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PHYSICS OF TECHNOLOGY

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HYDRAULIC DEVICES

Hydraulics and Equilibrium

HYDRAULIC DEVICES

A Module on Hydraulics and Equilibrium

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Hydraulic Devices

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Hydraulic Devices

INTRODUCTION

Hydraulic devices are used in many ways in our technical society. Many of these devices were developed in the nineteenth century, before electrical power was available. In those days the engineering work often involved applications of water and steam power. Hydraulic devices were an outgrowth of the technological needs of the times.

Today we still find many hydraulic (fluid-power) devices in our homes, hospitals, and industrial plants. Fluid-powered devices are relatively cheap, easy to maintain, and highly reliable; they are still very much a part of our technology. The basic physical principles of hydraulic devices are now well-known, so most hydraulic engineering today involves only the improvement of materials, methods of manufacture, and the development of new applications of fluid-operated devices. A few years ago an entirely new use of fluid technology resulted from the development of *fluidic amplifiers,* which sense and switch like their electronic counterparts. They are extremely reliable and rugged, so they are used in applications which would be too harsh for other kinds of amplifiers.

GOALS

In this module you will study the basic physics of fluids in order to understand some simple hydraulic devices. The goals of the module are outlined here. Read them before starting on the module, and refer to them as you work through the module.

The main goal of this module is to help you learn the basic principles and concepts of hydraulics. When you have completed the module, you should have a knowledge and understanding of the following:

1. Density of materials.
2. Properties of liquids.
3. The relationship between force and pressure.
4. The way in which pressure increases with depth in a liquid.
5. The way in which pressure is transmitted through fluids.
6. Pressure measurements.
7. The mechanical advantage and efficiency of hydraulic jacks.
8. The buoyant forces exerted on objects by liquids.
9. The way in which pressure depends on the rate of flow of a fluid.

SECTION A

HYDRAULIC DEVICES

A treatment of the properties of liquids is an important component of this module. You will study several devices in which the *static* (stationary) and *dynamic* (moving) properties of liquids are used to perform useful functions. The devices to be studied include automobile brakes, the hydraulic jack, the toilet tank, the siphon, the aspirator, the hydrometer, pressure gauges, and the sphygmomanometer.

In an automobile brake system, the brake fluid transmits pedal pressure to a brake shoe (or a pad for a disc brake). The brake shoe rubs against the brake drum and produces a frictional force to slow the auto. A 2-ton car, or a 10-ton truck, can be stopped by applying a force of a few pounds to the brake pedal. Why is such a small applied force able to stop a speeding car? What happens if there is air in the brake line? You will learn the answers to these and other questions as you proceed through the module.

The toilet tank uses a system of levers to release the water and a float-lever system to shut off the water supply after the tank has refilled. Did you ever watch the operation of the tank? What happens to shut off the water filling the tank?

The aspirator is a pump whose operation depends on the properties of a flowing liquid. You will see this device in operation in the laboratory.

The hydraulic jack has the effect of "magnifying" a force, so that a heavy object, such as a car, can be lifted with ease.

A siphon is a simple device (usually just a piece of tubing) which enables a liquid to flow from one container, over a high point, then down into a lower container, without the aid of a pump. It can be used to draw liquid from the lower part of a container when the surface is contaminated with scum, oil, or other unwanted matter. You might also use it to "borrow" gasoline from the tank of a car.

Figures 1 through 6 will serve as part of the basis for our discussions of fluid mechanics. You may need to look back at these illustrations as you work through the module.

So far we have used many technical terms without explaining their meaning. Some you already know, others are new to you. We will explain most of the terms to you as we come to them in the module.

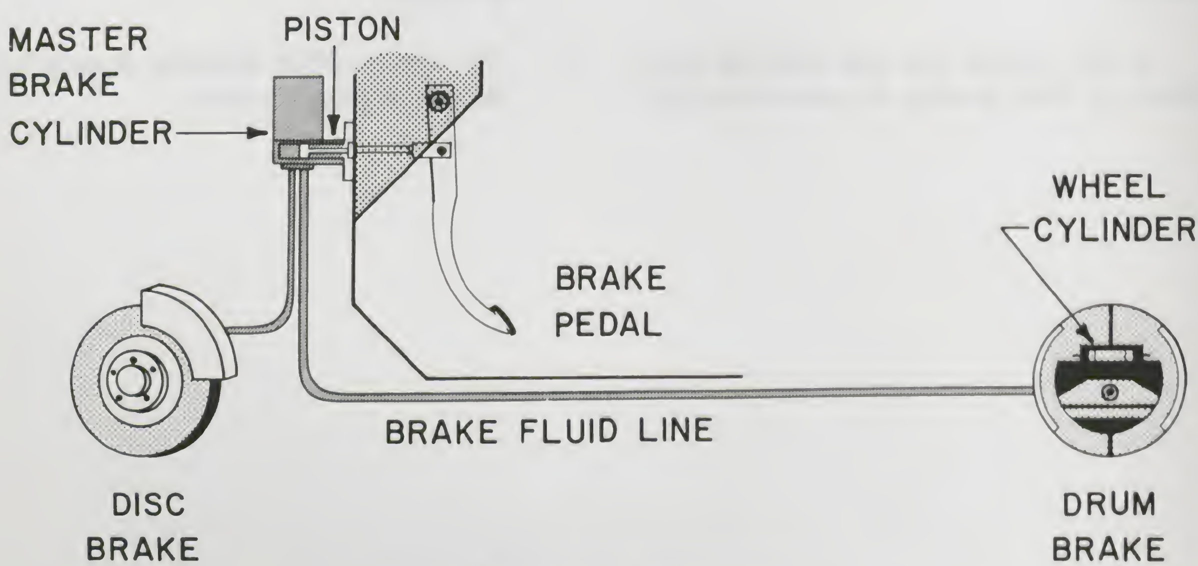


Figure 1. An automobile braking system.

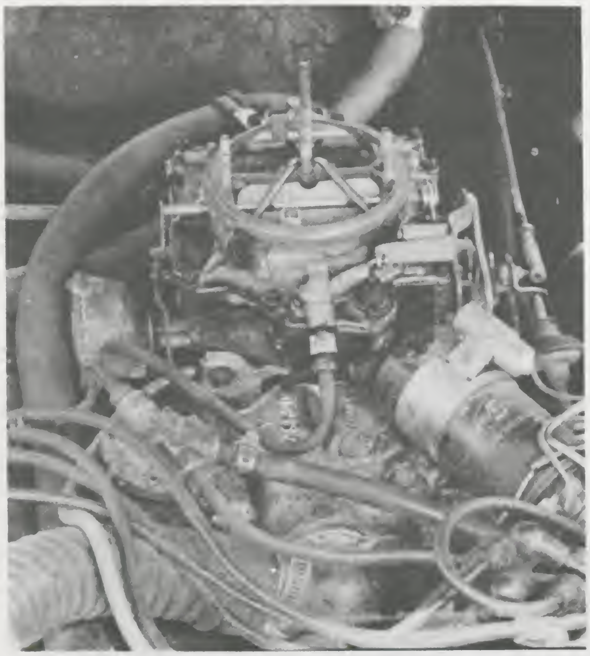


Figure 2. The carburetor is a device that makes use of the physical laws which apply to stationary fluids (*hydrostatics*) and moving fluids (*hydrodynamics*).

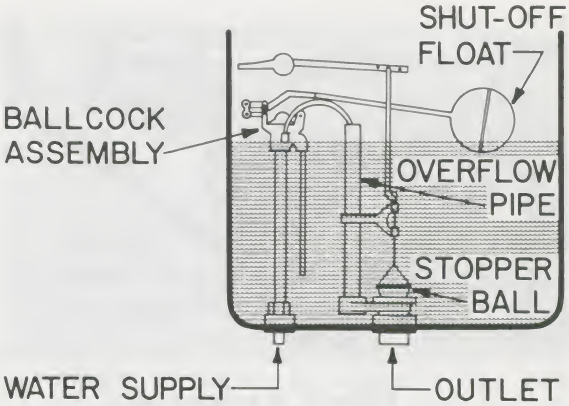


Figure 3. A toilet-tank mechanism.

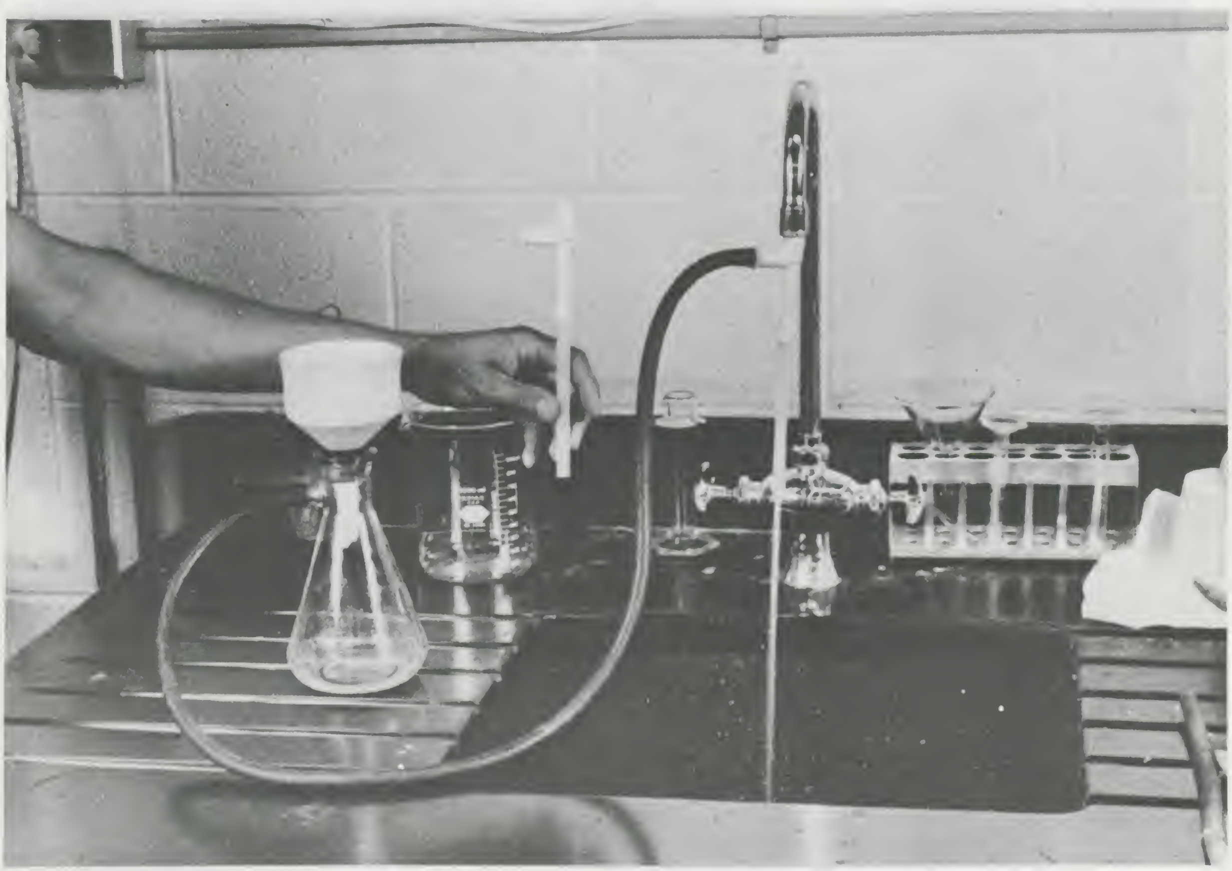


Figure 4. An aspirator being used in the lab.

Figure 5. A hydraulic jack.

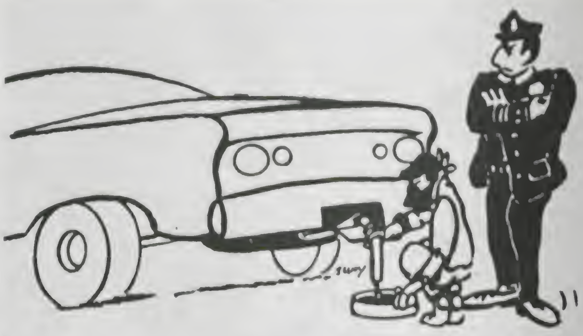


Figure 6A.

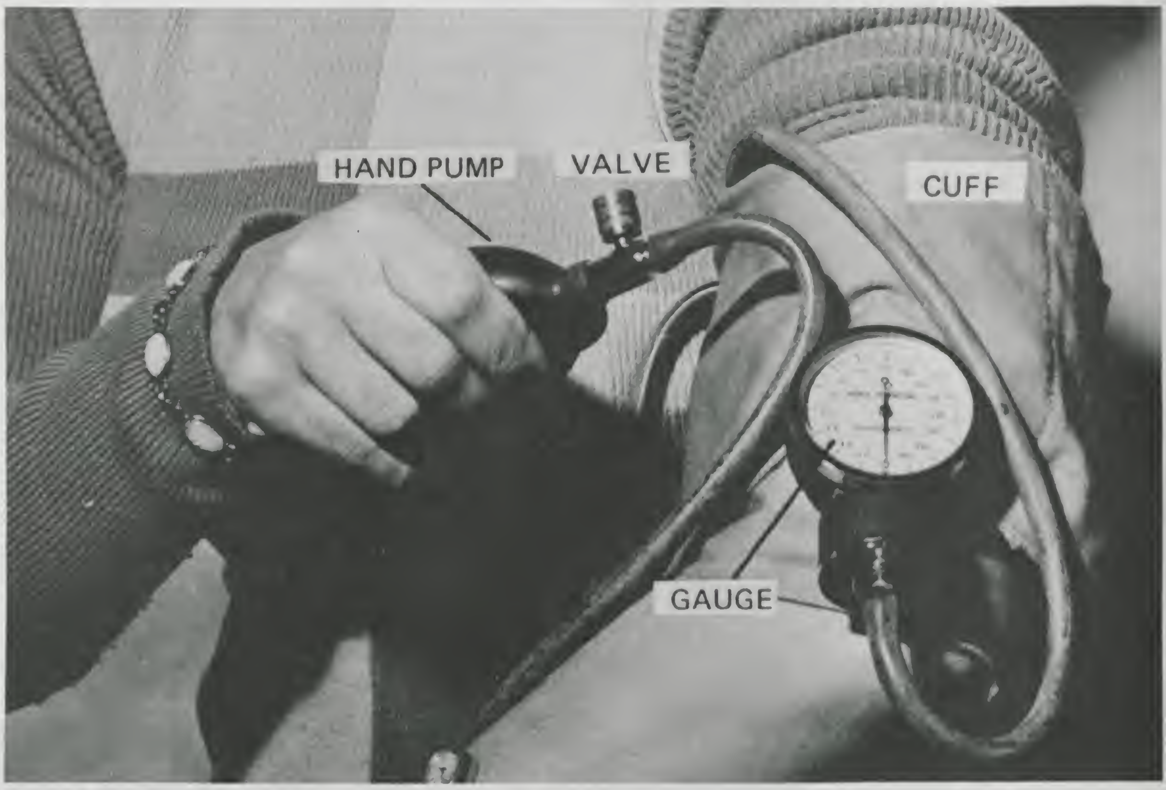


Figure 6B. A sphygmomanometer.

EXPERIMENT A-1. Devices

A. Auto Jack

Your instructor will provide you with a portable hydraulic jack that has a base plate and a load-lifting plate added to it. This device is a 1.5-ton-capacity jack of the type used to lift a car. Figure 7 shows this manually operated miniature garage jack.

Operate the jack; work it up and down. Use it to lift one student standing on the lift platform. Measure the force needed to move the handle to lift the student. Use a spring balance and read the dial as the handle moves downward. Fill in the worksheet for this experiment, which is found at the back of the module.

1. How many times greater is the lifted weight than the force applied to the handle? How shall we describe this device? Inexpensive? Reliable? Lifts heavy loads with a small applied force? Multiplies force?

2. Carefully look over the jack, then describe briefly what you see.

   Did you notice the two cylinders? The shaft from the smaller cylinder is attached to the bottom of the handle. The shaft from the larger cylinder transmits the lifting force to the object being lifted. Look for these now if you didn't find them before.

   Which cylinder is longer? The jack requires many full strokes of the handle to extend the shaft of the longer cylinder its full travel. Why?

3. What do you notice about the handle? Is it a lever? What is the approximate distance from the pivot to the other end? What is the approximate distance from the pivot to the shaft of the smaller cylinder? How many times bigger is the first length than the second length? Do you know the significance of this number? If not, you will soon.

   What's inside the jack? Can you guess? What reason do you have for your answer?

Figure 7.

This jack weighs less than 6 lb, costs about $12, and can lift 3000 lb with about 100-lb force applied to the handle. How does it work? The answer is described in this module. We will explain the behavior of fluids in terms of physical laws. When you understand those laws, the operation of this hydraulic jack and many other devices will be clear.

The hydraulic jack illustrates that a simple two-piston hydraulic system can be used for force multiplication.

B. A Two-Piston Demonstration System

Using the two-piston demonstration system (Figure 8) with valves and a pressure gauge (0-100 lb/in^2):

1. Demonstrate to yourself that the fluid is almost incompressible.

2. Observe that simple pressure-activated devices can be operated to control signals similar to the oil pressure light or brake lights in a car.

3. Push down on the smaller-diameter piston until you develop 80 lb/in^2 pressure in the system (you will have to close the valve between the gauge and the larger piston to build that pressure). Now, after adjusting the valves properly, push down on the larger-diameter piston to develop an 80 lb/in^2 pressure in the system. Which piston requires a larger force to produce this pressure? Can you explain why?

4. (Optional) Have a test of strength. Let the instructor press down on the large piston. Push down on the small piston—see who can push his piston all the way down.

5. (Optional) Place a 5-lb weight on the small piston. What pressure is developed in the system? Now do it with a 10-lb

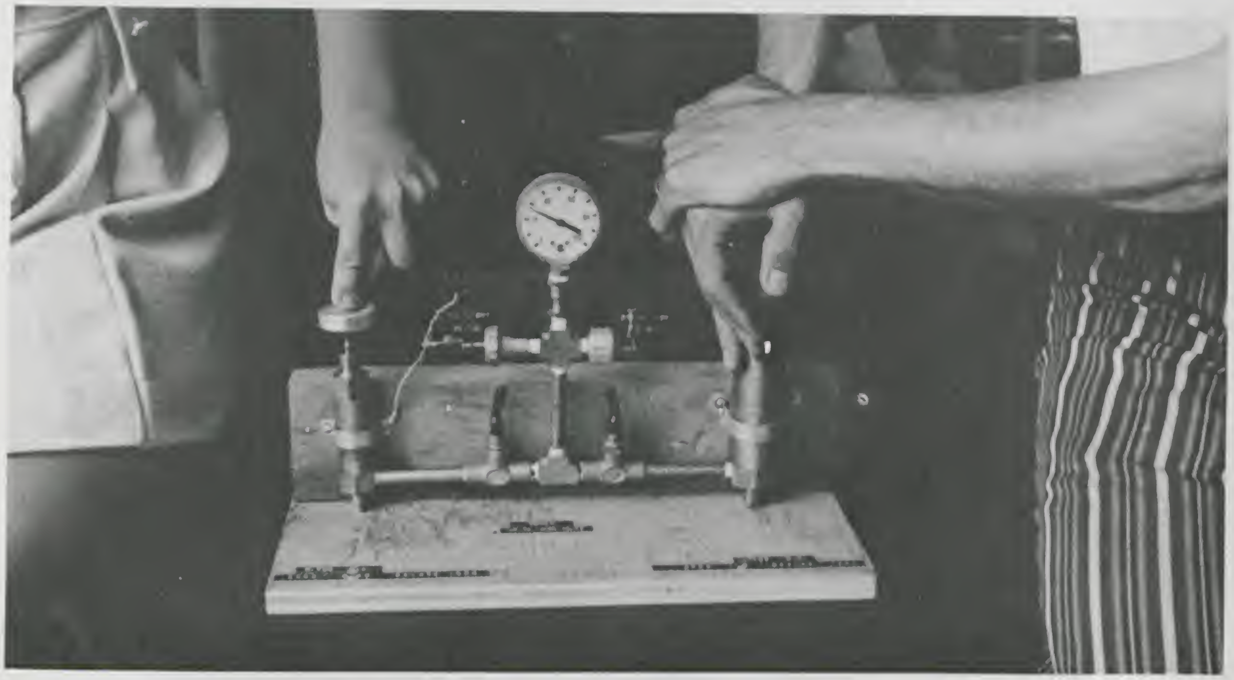


Figure 8. A two-piston hydraulic system.

and a 15-lb weight. Try the same weights on the larger piston. What pressures are produced? Can you explain the differences between the two sets of values?

6. In a hydraulic system, the amount of fluid stays the same if there is no leak. If the fluid is *incompressible,* then the volume it occupies also is constant when the pressure changes. This may seem quite obvious, but if the system were *pneumatic* (air operated), the volume would change as the pressure changed, since air is easily compressed. Using an empty syringe, demonstrate that the air inside is highly compressible by sealing the exit port with your finger while operating the plunger. You should be able to produce a substantial decrease in volume. What fraction of the original volume is the compressed volume? When you release the plunger, it should return almost to the starting point (keeping the exit port sealed). If it doesn't quite return, try to explain why and check your answer with your instructor.

C. Other Devices

Examine the other hydraulic devices displayed in the lab. As you think about what each one is for and how it operates, some unanswered questions may come to mind. For each device write down one or two questions you would like to have answered. Be sure to operate the aspirator and the siphon.

PROPERTIES OF LIQUIDS

The behavior of liquids has top billing in this module. It is important that we establish several of the basic properties of liquids. Some of these liquid properties will be very familiar, others will be new and interesting.

1. A liquid conforms to the shape of its container. This fact is one of the main differences between liquids and solids.

2. A liquid seeks its own level. When a liquid is poured into an open vessel, the surface of the liquid will reach the same level in all parts of the vessel.

Figure 9. A liquid reaches the same level in all parts of an open vessel.

3. Liquids are incompressible. No matter how hard you squeeze, you cannot *noticeably* change the volume of a sample of liquid. (There is a tiny bit of change at high pressures, but it can be neglected.) This is the most important difference between a liquid and a gas. A gas is easily compressed into a smaller volume. Because of its incompressibility, the ratio of the mass to the volume is a distinctive property of a given liquid. This useful and easily measured property is called the *mass density*. The Greek letter ρ (*rho*) is the commonly used symbol for mass density:

$$\rho = \frac{M}{V} \qquad (1)$$

where M = mass of the sample in kilograms (kg) and V = the volume of the sample in cubic meters (m^3). Mass density is measured in kg/m^3. Since this is a large unit, mass density is more often described in g/cm^3. $1\ g/cm^3 = 10^3\ kg/m^3$.

In the English system of units it is more common to use *weight density*, which is the ratio of weight to volume.

$$D = \frac{W}{V} \qquad (2)$$

where D is the weight density, W is the weight, and V is the volume. Typical units used for weight density are lb/ft^3 or ton/yd^3. Table I lists density values for several common substances. Several solids are included, even though this module emphasizes liquids.

A word of caution: the density of a material depends on its temperature. Because the volume of a substance usually changes when its temperature changes, while the mass and weight remain unchanged, the density also changes. To be precise, one should always give temperature data along with the density.

Weight density and mass density are related because the weight of an object is related to its mass:

$$W = Mg$$

where g is the acceleration due to gravity ($9.8\ m/s^2$ or $32\ ft/s^2$). Therefore, the weight density is simply related to the mass density by

$$D = \frac{M}{V}g = \rho g \qquad (3)$$

Table I.

Densities of Several Materials at Room Temperature

	Mass Density		Weight Density
LIQUIDS	(g/cm^3)	(kg/m^3)	(lb/ft^3)
Water	1.00	1,000	62.4
Alcohol	0.79	790	49.4
Maple Syrup	1.4*	1,400*	87.5*
Ethylene Glycol (Antifreeze)	1.12	1,120	70.0
Glycerine	1.26	1,260	78.8
Turpentine	0.87	870	54.4
Mercury	13.5	13,500	843.8
Olive Oil	0.91	910	56.9
SOLIDS			
Iron	7.9	7,900	493.8
Copper	8.9	8,900	556.3
Aluminum	2.7	2,700	168.8
Gold	19.4	19,400	1206.3
Glass	2.6*	2,600*	162.5*
Bone	1.8*	1,800*	112.5*
Diamond	3.2*	3,200*	200.0*
Wood	0.7*	700*	43.8*

*Typical value

Example 1. A sample of metal has a volume of 0.10 m^3 and a mass of 790 kg. Determine its density, and check Table I to see what material it might be.

Solution. Because mass is specified, we can most readily determine mass density.

$$\rho = \frac{M}{V}$$

$$= \frac{790 \text{ kg}}{0.1 \text{ m}^3}$$

$$= 7.9 \times 10^3 \text{ kg/m}^3 = 7.9 \text{ g/cm}^3$$

From the table we see that this is the density of iron.

Example 2. How many tons of water are there in a swimming pool 15 ft wide, 30 ft long, and 4 ft deep?

Solution. The weight can be determined from the volume of the pool and the weight density of water:

$$D = \frac{W}{V}$$

$$W = DV$$

The volume of the pool can be determined from the dimensions:

$$V = (15 \text{ ft})(30 \text{ ft})(4 \text{ ft})$$

$$= 1800 \text{ ft}^3$$

From Table I, the weight density of water is 62.4 lb/ft^3.

$$W = (62.4 \text{ lb/ft}^3)(1800 \text{ ft}^3)$$

$$= 112{,}000 \text{ lb}$$

$$\cong 56 \text{ tons}$$

Sometimes a material is described by its *specific gravity*. The specific gravity is a number which compares the density of the substance to the density of water.

$$\text{Specific gravity} = S.G. = \frac{\text{density of substance}}{\text{density of water}}$$

Specific gravity is a dimensionless number. Since the density of water is 1.0 g/cm^3, we can easily calculate the density of any substance if we know its specific gravity.

Example 3. By law, maple syrup must meet minimum density standards. The specific gravity of a sample of maple syrup is 1.412. What is its density?

Solution.

$$S.G. = \frac{\text{density of syrup}}{\text{density of water}}$$

$$1.412 = \frac{\text{density of syrup}}{1.0 \text{ g/cm}^3}$$

$$\text{density of syrup} = 1.412 \text{ g/cm}^3$$

One practical use of specific gravity is in the measurement of the "proof" of alcoholic solutions, as Table II shows. Notice that the temperature is specified.

There are many other practical uses for measurements of specific gravity. For example, since the specific gravity of the acid in a car battery depends on the condition of charge, the battery can be tested simply by measuring that specific gravity. Likewise, the percentage of antifreeze in the cooling system can be determined through a specific-gravity measurement. The specific gravity of a liquid can be measured easily by using a simple instrument called a *hydrometer*.

Table II.

Alcohol Strength and Specific Gravity,
As Measured in the United States

All measurements are made at a temperature of 60°F.

Percent by volume	Proof	Specific gravity
0.0	0.0	1.0000
5.0	10.0	0.9928
10.0	20.0	0.9866
15.0	30.0	0.9810
20.0	40.0	0.9759
30.0	60.0	0.9653
40.0	80.0	0.9517
50.0	100.0	0.9342
60.0	120.0	0.9133
70.0	140.0	0.8899

(Table courtesy of Taylor Wine Co., Hammonds-Port, N. Y.)

EXPERIMENT A-2. Hydrometry

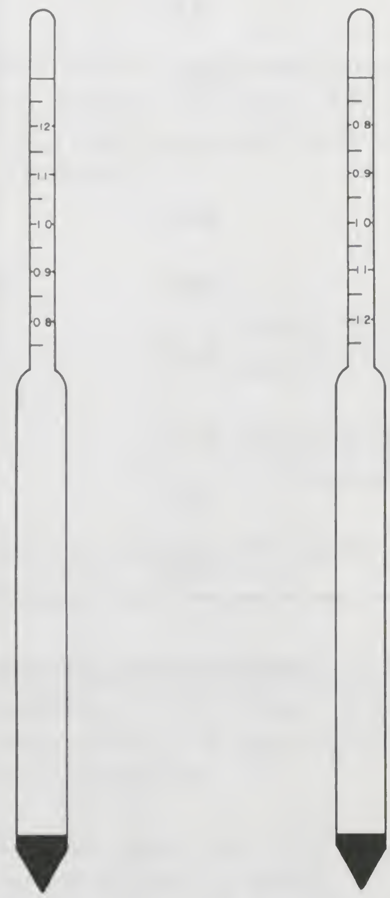
A. A *hydrometer* is a simple device for measuring the specific gravity of a liquid. Hydrometers are used in the chemical industry for process-control and testing. They are used in the maple syrup, wine, whiskey, and milk industries for quality control tests.

In its simplest form, the hydrometer is a rod of wood, or a hollow glass tube, weighted slightly at one end so that it floats in an upright position. A scale is usually marked on it. Each hydrometer, of course, has a certain fixed mass. When floated in a beaker of water, it will sink to a certain depth. The water line is marked 1.0 on the hydrometer, since this is the specific gravity of water. If the hydrometer is placed into any other liquid and it comes to rest with the liquid surface at the 1.0 line, then that liquid also has a specific gravity of 1.0 and a density of 1 g/cm^3.

Figure 10. A hydrometer.

Question 1. Suppose you have a beaker of liquid which is more dense than water. That is, it has a higher specific gravity than water. Each cubic centimeter has mass greater than one gram. Will the hydrometer sink deeper into the new liquid than into water?

Question 2. Which hydrometer of Figure 11 has the right scale? Why?



(A) (B)

Figure 11.

1. Obtain a beaker, a hydrometer, and several different liquids from your instructor, and measure the specific gravity of each liquid. Compare your answers to the known specific gravity of the liquids supplied. You may have to look up the known specific gravity in a book of tables, such as the Chemical Rubber

Company's *Handbook of Chemistry and Physics*.

2. Your instructor may also ask you to identify unknown liquids by their specific gravities.

3. Heat some water in a beaker. With a hydrometer in the beaker, slowly add and dissolve sugar into solution. What range of specific gravity do you measure? Compare your results to those in a handbook of physics or some other reference source for density values (the handbook may list *sucrose,* which is the chemist's name for sugar).

Question 3. The average human body has a specific gravity of 0.95. Should it float in fresh water? How would it float differently in sea water? What should happen when a floating person takes a deep breath or exhales deeply?

B. The freezing point of pure water is 32°F. Adding impurities lowers the freezing-point temperature. Usually *ethylene glycol* is added as antifreeze to accomplish this. A table showing mixture ratios for water and antifreeze to provide protection is often supplied on the antifreeze container. If one wishes to check the antifreeze content already in a radiator, he can use a hydrometer and look up the freezing point that corresponds to the determined specific gravity. The cheapest do-it-yourself hydrometers do not have a continuous specific gravity scale. Instead, they utilize four balls which have different densities. Each one just floats when the specific gravity of the solution matches that of the ball. The number of balls which float indicates the specific gravity, thus the freezing point of the radiator fluid. Typically, the balls are designed to float at concentrations of antifreeze which produce freezing points of +30, +15, zero, and −15°F.

1. Using an automotive hydrometer, a graduated cylinder, some ethylene glycol, and water, determine the density of each of the balls in the inexpensive hydrometer. When a ball just floats, its density is equal to the density of the antifreeze + water mixture. The density of the mixture then corresponds to the freezing temperature indicated on the hydrometer scale. To start put 50 cm^3 of water into a 125-cm^3 graduated cylinder. Add ethylene glycol in 5-cm^3 increments. In the table at the back of the module, record the volume of the solution. Test after each increment of antifreeze to see how many balls float. Note the mixture when each ball floats.

   The density of the liquid at any increment of antifreeze is given by:

$$\rho_{mix} = \frac{\text{total mass}}{\text{total volume}} = \frac{V_W \rho_W + V_A \rho_A}{V_T}$$

   where:

   V_W, the volume of water, was set at 50 cm^3

   V_A, the volume of antifreeze, is increased in 5-cm^3 increments

   ρ_W = density of water = 1.0 g/cm^3

   ρ_A = density of antifreeze = 1.125 g/cm^3

2. Calculate solution density (and ball density) when each ball just barely floats.

3. Compare the mixture found experimentally for each degree of protection with the antifreeze-to-water ratio suggested on a container of ethylene glycol permanent antifreeze for the same protection.

SUMMARY

A liquid conforms to the shape of its container.

Liquids are nearly incompressible.

Mass density ρ is the ratio of mass M to volume V for a substance:

$$\rho = \frac{M}{V}$$

Weight density D is the ratio of weight W to volume V for a substance:

$$D = \frac{W}{V}$$

Specific gravity $S.G.$ is a dimensionless number:

$$S.G. = \frac{\text{density of a substance}}{\text{density of water}}$$

PROBLEMS

If you understand the module so far, then you should be able to do the following problems. Be sure to show the correct units with each numerical answer.

1. A block of wood weighs 160 lb and has dimensions of 2 ft $\times$ 2 ft $\times$ 1 ft. Calculate the weight density of the block in lb/ft^3.

2. What is the volume of 500 g of gold?

3. A vat of maple syrup contains 1600 gallons of syrup. Calculate the weight of the syrup in the vat. (7.5 gal = 1 ft^3)

4. The specific gravity of a metal alloy is 2.62. Calculate the mass density and the weight density of the alloy.

5. The specific gravity of an oil sample is 0.85. What is the volume of 40,000 lb of this oil?

6. A jet plane loads 14,000 lb of fuel, specific gravity 0.9. How many gallons of fuel does the plane load? (7.5 gal = 1 ft^3)

7. A government agent confiscates 6 kg of a smuggled narcotic. If the substance just fills a 5-liter container, what is its density in g/cm^3? (1 liter = 10^3 cm^3)

SECTION B

PRESSURE

Pressure is a ratio of force to area. In liquids it is usually easier and more meaningful to measure pressures than forces. If a force F acts over an area A and is perpendicular (*normal*) to the surface, the pressure is:

$$P = \frac{F}{A} \tag{4}$$

Example 4. A cylinder, whose base has an area of 5 in^2, contains 10 lb of water. What is the pressure on the bottom due to the water?

Solution. We can apply the definition of pressure, noting that the force is simply the weight of the water and that it is normal to the bottom of the cylinder.

$$P = \frac{F}{A}$$

$$= 10\ \text{lb}/5\ \text{in}^2 = 2\ \text{lb/in}^2$$

Example 5. An automobile radiator cap is designed to release steam if the pressure reaches 5 lb/in^2. At that pressure, how much force acts on the cap if it is sealing an opening 1.5 in in diameter?

Solution.

$$P = \frac{F}{A}$$

Therefore,

$$F = PA$$

Since $A = \pi d^2/4$, where d is the diameter of the circle,

$$F = (5\ \text{lb/in}^2)\frac{\pi d^2}{4}$$

$$= \frac{\pi}{4}(5\ \text{lb/in}^2)(1.5\ \text{in})^2$$

$$= 8.8\ \text{lb}$$

Example 6. A stack of books whose total mass is 10 kg rests on a table top. The bottom book has an area of 300 cm^2. Determine the pressure in N/m^2 exerted on the table.

Solution. The force is the weight of the object:

$$W = F = Mg$$

$$F = (10\ \text{kg})(9.8\ \text{m/s}^2)$$

$$= 98\ \text{kg}\cdot\text{m/s}^2 = 98\ \text{N}$$

$$P = \frac{F}{A} = \frac{98\ \text{N}}{300\ \text{cm}^2} = .33\ \text{N/cm}^2$$

However, we want to calculate the pressure in N/m^2, so we must convert from square centimeters to square meters.

$$1\ \text{m} = 10^2\ \text{cm}$$

$$1\ \text{m}^2 = (10^2\ \text{cm})^2 = 10^4\ \text{cm}^2$$

$$P = \frac{.33\ \text{N}}{\text{cm}^2} \times \frac{10^4\ \text{cm}^2}{\text{m}^2}$$

$$= 3300\ \text{N/m}^2$$

Although one commonly encounters pressure specified in English units (lb/in^2), the SI unit is the metric unit (N/m^2), which is called a Pascal (Pa).

$$1\ \frac{\text{N}}{\text{m}^2} = 1\ \text{Pa}$$

EXPERIMENT B-1. Pressure Dependence on Depth

Pressure in a liquid is related to the depth below the surface, as any swimmer will readily tell you. This fact is the design basis of many water distribution systems. Towns and villages pump their water up into high water towers to assure adequate pressure everywhere in their area. In cities, tall buildings have their own water towers raised on platforms above the roof. Why is that done? Often, automatic fire sprinkler systems have a separate water tank on the roof to assure adequate pressure on all floors. Nuclear power reactors have high tanks of water to assure emergency reactor core cooling should an accident occur. You may have observed that in your own house the pressure upstairs is less than the pressure downstairs.

In this experiment you will use a simulated water stand-pipe to determine the pressure as a function of depth for the water in the pipe.

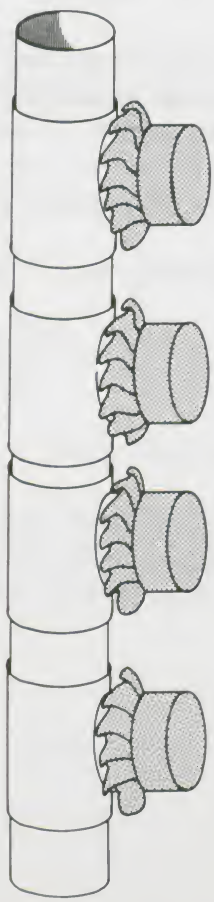


Figure 12. A stand-pipe simulator.

In many engineering investigations it is too difficult or too expensive to build and test a real system. The engineer may build a model or a simulator. In this context, a *model* means a smaller, scaled replica of the real system. By a *simulator* one means a small system that will behave similarly to the real system even if it is not an exactly scaled-down copy. This experiment will be done using a simple simulation of a building water system, as shown in Figure 12, to show you the effects of pressure on depth. You might consider each port to correspond to a floor. A real system would be about twelve times as high, or approximately 60 ft tall. Real pressures would be about twelve times what you will measure.

Here you will measure the pressure as a function of depth by employing the spring push balance as shown in Figures 13 and 14. Be sure the stand-pipe is full to the top before beginning.

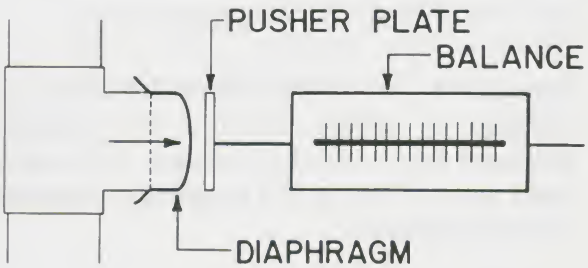
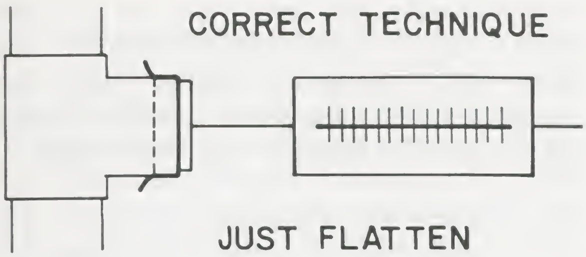


Figure 13.

1. Carefully measure the force necessary to flatten the flexible diaphragm at each surface and record your answers in the worksheet at the end of the module. It helps to have one person align the pusher plate while another person reads the compression balance.

   Note that the balance is calibrated in mass units (grams). To convert to force units (dynes) one should multiply by g, the acceleration due to gravity. Thus

$$P = \frac{F}{A} = \frac{Mg}{A}$$



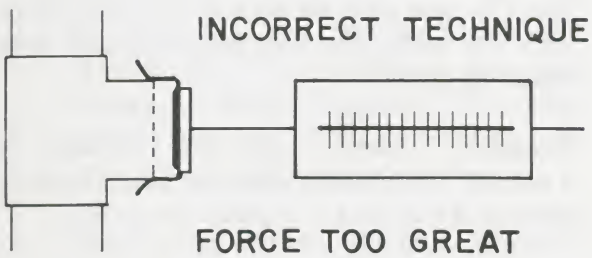


Figure 14.

where

$$g = 980 \text{ cm/s}^2$$

2. The area of the pusher plate is determined by measuring the diameter and using the formula:

$$A = \frac{\pi d^2}{4}$$

where

A = area of a circle

d = diameter

$\pi \cong 3.14$

Calculate the pressure P for each port. Measure and record the distance h to the center of each port from the water surface.

3. Plot on graph paper P versus h. Use a full page. Draw the straight line which best fits your data points. (If you don't know how, ask your instructor for help.) If any point seems very far from the line, recheck your data. The pusher plate may have been hitting the metal wall of the port.

4. The *slope* of a line on a graph is the *rise* (the vertical distance between two points) divided by the *run* (the horizontal distance between the same two points). By taking measurements from your graph between any two points on the line (pressures P_1 and P_2 and depths h_1 and h_2 respectively) find the slope of the graph. The worksheet at the back of the module shows you how to do this step by step. You will use this value later.

PRESSURE VERSUS DEPTH

We will now develop an expression for the way pressure varies with depth and see if it corresponds to the results of your experiment. Figure 15 represents a column of fluid of depth h and cross-sectional area A.

The volume of the fluid is

$$V = Ah$$

The weight of the fluid is

$$W = DV \tag{5}$$

The column of fluid rests on an area A, so the pressure at the base of the column due to the fluid is

$$P = \frac{F}{A} = \frac{W}{A} = \frac{DV}{A} = \frac{DAh}{A}$$

Or

$$P = Dh \tag{6}$$

In words, the pressure beneath the surface of a liquid is equal to the product of

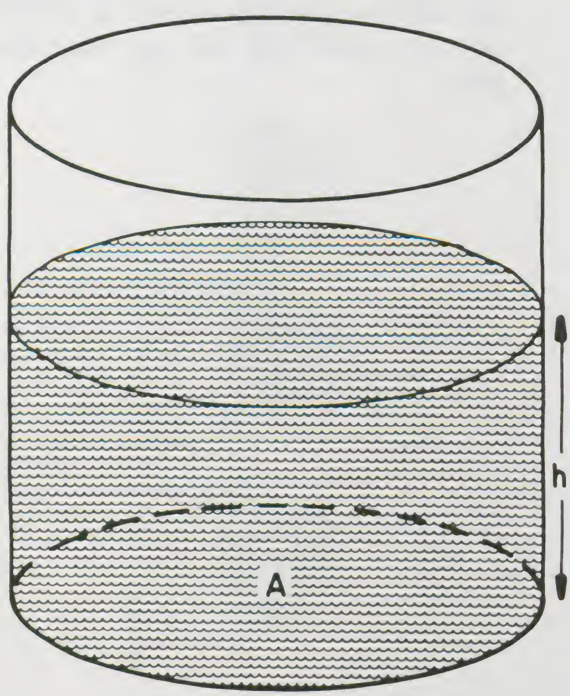


Figure 15.

weight density (D) and depth (h). D is the *slope* of the P versus h graph. How does the slope you measured compare with the accepted value of the weight density of water ($D = 1\ \text{g/cm}^3$)? Since $D = \rho g$, we can write

$$P = \rho g h \tag{7}$$

When using either Equation (6) or Equation (7), one must be careful to be consistent with the units. The following examples illustrate this point.

Example 7. Maple syrup has a density of $1.4\ \text{g/cm}^3$. Find the pressure in a large tank of syrup at a depth of 1 m below the surface.

Solution. Because mass density is specified, Equation (7) is appropriate. A consistent set of units is obtained if the depth is expressed in centimeters.

$$P = \rho g h$$

$$= (1.5\ \text{g/cm}^3)\ (980\ \text{cm/s}^2)\ (100\ \text{cm})$$

$$= 1.47 \times 10^5\ \frac{\text{g}\cdot\text{cm/s}^2}{\text{cm}^2}$$

$$= 1.47 \times 10^5\ \frac{\text{dyn}}{\text{cm}^2}$$

Example 8. Determine the pressure on a diver when he is 30 ft below the surface of a fresh water lake.

Solution. The weight density of water is 62.4 lb/ft^3.

$$P = Dh$$

$$= (62.4\ \text{lb/ft}^3)\ (30\ \text{ft})$$

$$= 1870\ \text{lb/ft}^2$$

SOME FACTS ABOUT PRESSURE

1. An interesting feature of Equations (6) and (7) is that there is nothing in either equation to specify the *shape* (geometry)

of the liquid column. In fact, if a series of differently shaped containers are all attached to pressure gauges and filled to the same level with the same liquid, the pressures are seen to be the same. That is, the pressure at any given depth is the same no matter what the shape of the container.

2. In addition, the pressure does not depend upon the orientation of a solid surface on which it might act. No matter which way the surface is tilted, the pressure at a given depth is the same.

3. Finally, no forces act in a direction parallel to a solid surface which is under the surface of a stationary liquid. That is, the forces produced by pressure are always perpendicular (normal) to the surface.

TALL BUILDINGS

In Experiment B-1 you probably noticed that the pressure at the lowest port was about four times the pressure at the highest port. Does this agree with your own experience in tall buildings? If real buildings worked exactly this way, some very high pressures would be found on the lower floors. For example, the Empire State Building is 102 stories (about 1100 ft) high. If the water pressure at the upper observation deck was 15 lb/in^2, then in the lobby the pressure would be about 1530 lb/in^2! Imagine how thick the pipe walls might have to be to contain that. If the water pressure at the top floor is adequate, engineers install a device, called a *pressure regulator,* at each floor. It keeps the pressure from getting too high at that floor. A two- or three-story house doesn't usually have pressure regulators, so you can often observe pressure differences there.

Question 4. The water pressure in the top floor of a factory is 60 lb/in^2. How high above the floor must the top of the water tower be to produce this pressure?

ARCHIMEDES' PRINCIPLE

We have spent some time discussing the properties of liquids—now we shall concern ourselves with the behavior of objects that are placed in liquids, i.e., things that sink or float.

Why does a 40,000-ton steel ship float? Exactly how big must the diving tanks on a research submarine be so that it can explore the bottom of the ocean? Today many river barges are made from poured concrete. Doesn't concrete sink when you throw it in water? Why does a heavy rock that you lift from the bottom of a lake feel heavier as it clears the water surface? The ball float in a toilet tank and the float in your car's carburetor experience forces that keep them at the surface of a liquid. What causes those forces?

In Experiment A-2 did you observe that a hydrometer that floats at some level in one liquid floats deeper in a second liquid and perhaps sinks to the bottom in a third liquid? In some way, the density of the liquid determines whether an object will float or sink. The Greek philosopher Archimedes had a clear understanding of the buoyancy (floatability) of objects over 2000 years ago. His discoveries have influenced technology for 20 centuries. The following is a statement of *Archimedes' Principle*:

> An object placed in a liquid experiences an upward force equal to the weight of the liquid it displaces.

An illustration of this is shown in Figure 16. Each cubic foot of liquid that is displaced causes the metal cylinder to apparently "lose" 50 lb of weight. The buoyant force is equal to the weight of the displaced fluid. (This liquid isn't water, which weighs 62.4 lb/ft^3.)

When the metal cylinder is suspended this way, the spring balance reads its *apparent weight*. The apparent weight is the true weight minus the buoyant force exerted by the liquid. By Archimedes' Principle, that buoyant force is equal to the weight of the displaced liquid.

Figure 16 shows the displaced liquid spilling out of the full container as the metal cylinder is lowered into it. Note, however, that

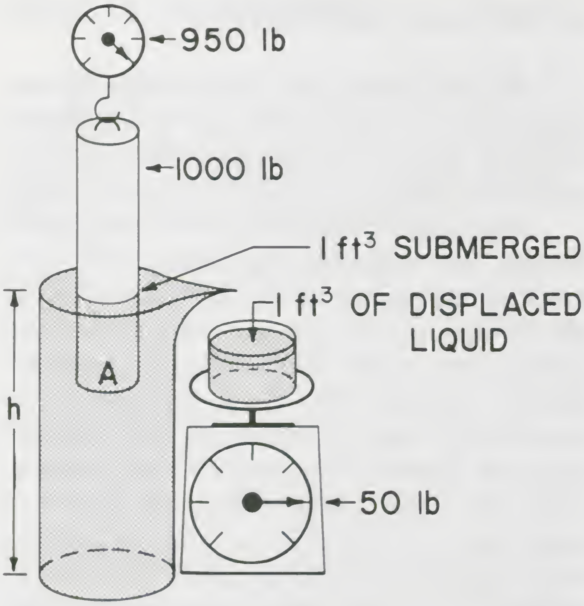


Figure 16.

the displaced fluid doesn't *have* to leave the container in order for the cylinder to experience a buoyant force. "Displaced" means that the liquid is moved aside to make room for the object. If the cylinder were lowered into a partially full container, the displaced liquid would just rise in the container.

If a 125-lb piece of wood floats on the surface of a lake, it displaces 2 ft^3 (125 lb) of water. Its apparent weight is zero. In general, an object floats if it displaces a weight of liquid equal to its own weight. An additional downward force is necessary to completely submerge a floating object.

The size of a large ship is usually characterized by its *deadweight displacement,* in tons. A 125,000-ton displacement oil tanker displaces 125,000 tons of sea water when it is fully loaded. The tanker floats when loaded because it weighs the same as do 125,000 tons of sea water.

DERIVATION OF ARCHIMEDES' PRINCIPLE

We can show that the upward or buoyant force on a submerged object is equal to the weight of the displaced liquid by analyzing Figure 17. Recall that a liquid exerts a normal (perpendicular) force on any surface in the liquid.

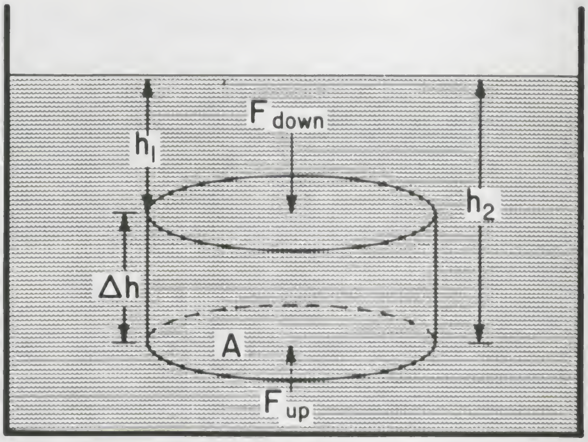


Figure 17.

When a cylinder, such as the one in Figure 17, is submerged in a liquid, the pressure on the top surface is:

$$P = Dh_1 = \frac{F_1}{A}$$

where D is the weight density, A is the cross-sectional area of the cylinder, and F_1 is the total force on the top surface. The force on the top of the submerged cylinder is thus

$$F_1 = Dh_1 A$$

Similarly, the force on the bottom is $F_2 = Dh_2 A$. Since h_2 is greater than h_1, F_2 is greater than F_1. There are also forces acting on the sides of the cylinder, but for each such force on one side there is an oppositely directed matching force on the other side. Thus all the horizontal forces cancel out and the net force on the cylinder is

$$F_B = F_2 - F_1$$

$$= DA\,(h_2 - h_1) = DA\Delta h$$

The buoyant force, F_B, is the net force of the liquid on the object. The volume of the displaced fluid is $V = A\Delta h$. Therefore, $F_B = DV$, which is the weight of the displaced fluid. We have derived Archimedes' Principle for a solid whose volume is easy to calculate. The same analysis can be applied to other shapes as well, although it is more difficult to analyze complicated shapes.

The shut-off valve in a toilet tank illustrates an application of Archimedes' Principle. It relies on the buoyant force on a floating ball to shut off the valve.

Example 9. A cubic foot of iron is totally immersed in water.

a. Determine the buoyant force on the iron.

b. Determine the apparent weight of the iron.

Solution. a. Archimedes' Principle states that the buoyant force is equal to the weight of the displaced fluid. One cubic foot of water is displaced. It weighs 62.4 lb. Therefore, the buoyant force is 62.4 lb.

b. The apparent weight is the true weight minus the buoyant force. The true weight is the product of the weight density of the iron and the volume.

$$W = DV$$

$$= (494 \text{ lb/ft}^3)\ (1 \text{ ft}^3)$$

$$= 494 \text{ lb}$$

Apparent weight = 494 lb − 62.4 lb = 431.6 lb.

Example 10. A wood float has a density of 600 kg/m^3. Its mass is 2 kg.

a. Determine how much of its volume is submerged when it floats in water.

b. Determine the force required to pull it below the surface.

Solution. a. The float will sink until it displaces an amount of water equal to its own weight, $W = Mg$. Therefore, the *mass* of the displaced water is 2 kg. The volume of displaced water can be determined from the mass density of water.

$$V = \frac{M}{\rho} = \frac{2 \text{ kg}}{1 \text{ g/cm}^3} = \frac{2000 \text{ g}}{1 \text{ g/cm}^3}$$

$$= 2000 \text{ cm}^3$$

$$= 2 \times 10^{-3} \text{ m}^3$$

b. The force required to pull the float below the surface is the difference between the buoyant force on the completely submerged float and the true weight of the float. First, using the density of the wood, determine the volume of the wood block, which is also the volume of water displaced when the wood is completely submerged.

$$V = \frac{M}{\rho} = \frac{2 \text{ kg}}{600 \text{ kg/m}^3} = 3.3 \times 10^{-3} \text{m}^3$$

The buoyant force equals the weight of the displaced water.

$$F_B = \rho g V$$

$$= (10^3 \text{ kg/m}^3) \times (9.8 \text{ m/s}^2) \times$$

$$(3.3 \times 10^{-3} \text{m}^3)$$

$$= 32.3 \text{ N}$$

The weight of the float, Mg, is 19.6 N. Subtracting, the required force is 12.7 N.

Now you should be ready for Experiment B-2, an experiment which uses Archimedes' Principle in the determination of densities.

The simplest way to determine the density of an object is to measure its mass and volume. One can calculate the volume if the object has a regular shape, as do a sphere, a cube, and a cylinder. The density is then calculated by taking the ratio of the mass to the volume:

$$\rho = M/V$$

If the object is irregularly shaped, it may be difficult or impossible to find the volume by measuring its dimensions.

A. Finding Density from Displaced Volume

Here is a method that can be used with small objects which are irregularly shaped.

1. Determine the mass of the object with a gram balance.

2. Partially fill a graduated cylinder with water and record the volume of water in cubic centimeters.

3. Place the object in the cylinder. If it is more dense than the water, it will sink and displace a volume of water equal to its own volume so that the water rises in the cylinder. If it is less dense than water, you will have to hold it just under the water with a stiff wire which does not displace much water.

4. Record the volume (water plus object) in the graduated cylinder. The difference between the two readings of the water level is the volume of the object.

5. Take the mass-to-volume ratio to obtain the density, ρ_x, for each object.

Perform this experiment for each of the objects provided by your instructor.

B. A More Accurate Measure of Density

Your measurement of mass is quite accurate because the balance is a sensitive device. You should be able to determine an object's mass to the nearest 0.1 g. However, your measurement of volume using the graduated cylinder is a relatively crude measurement; the graduated cylinder does not give a precise measure of volume. Thus, using this method to measure volume isn't a very good way to determine density. It is better to measure volume more accurately. One way to do this is by using the gram balance.

Using Archimedes' Principle, a balance can be used as illustrated in Figures 18 and 19.

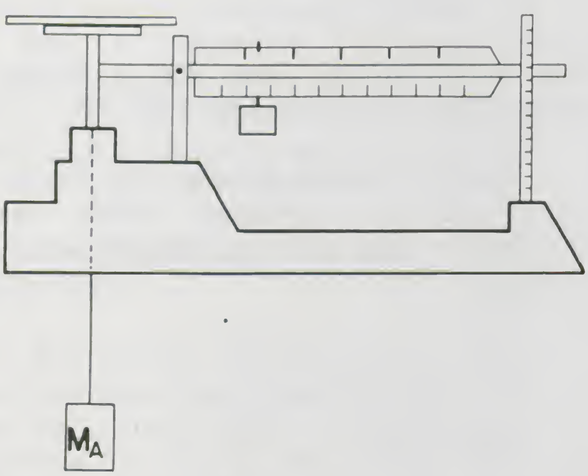


Figure 18.

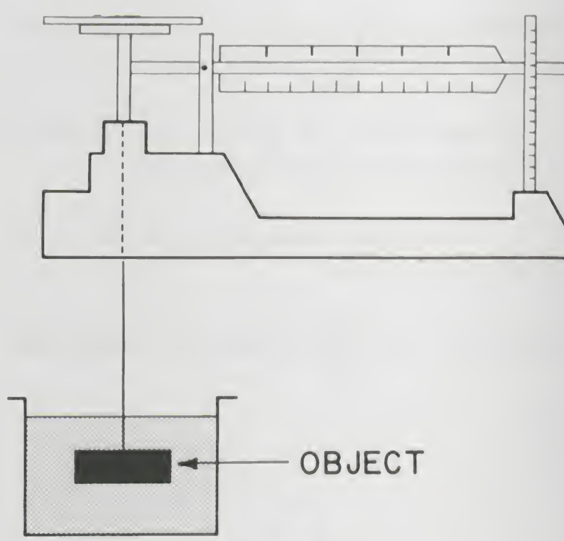


Figure 19.

1. With the balance supported over the table, hang the object in question from the balance arm, as shown in Figure 18, and find its mass in air, M_A.

2. Now suspend the object from the balance arm so that it is submerged in a beaker of water, as in Figure 19. Be careful that it doesn't touch the sides. Record the apparent mass in water, M_W.

The difference between these two values (M_A - M_W) is related to the apparent loss in weight, which is in turn equal to the buoyant force exerted by the water. By Archimedes' Principle, that force is just equal to the weight of the water displaced. Thus, the weight of the water displaced is just the weight of (M_A - M_W), in grams (i.e., $M_{Water} = M_A - M_W$). How many cubic centimeters of water are there in (M_A - M_W) grams of water? Use the density relationship and the density of water (1 g/cm^3):

$$\rho_{Water} = \frac{M_{Water}}{V} = \frac{(M_A - M_W)}{V} = 1 \text{ g/cm}^3$$

Or:

$$V = (M_A - M_W)/(1 \text{ g/cm}^3)$$

Thus, the volume (in cm^3) of the object *is numerically equal* to the apparent loss in mass represented by (M_A - M_W). By making two readings of mass you have obtained the volume of the object to a much higher accuracy than by using the graduated cylinder. Using the expression above for the volume, you can now compute the density, ρ_x, of the object:

$$\rho_x = \frac{M_A}{V} = \frac{M_A}{(M_A - M_W)/(1 \text{ g/cm}^3)}$$

Or:

$$\rho_x = \frac{M_A}{M_A - M_W} \times (1 \text{ g/cm}^3)$$

3. Using this method, repeat the determination of ρ_x for each of the objects previously used.

C. Objects Which Float

If you wish to use this latter method to find the density of an object which floats, you must use a sinker to pull it under the water.

1. Measure the mass in air, M_A, of the object by hanging it from the balance arm as before.

2. Suspend a heavy metal sinker from the object, as shown in Figure 20.

3. Submerge the sinker, as in Figure 20, and measure and record the apparent mass of the object in air plus the sinker in the water ($M_A + M_{SW}$).

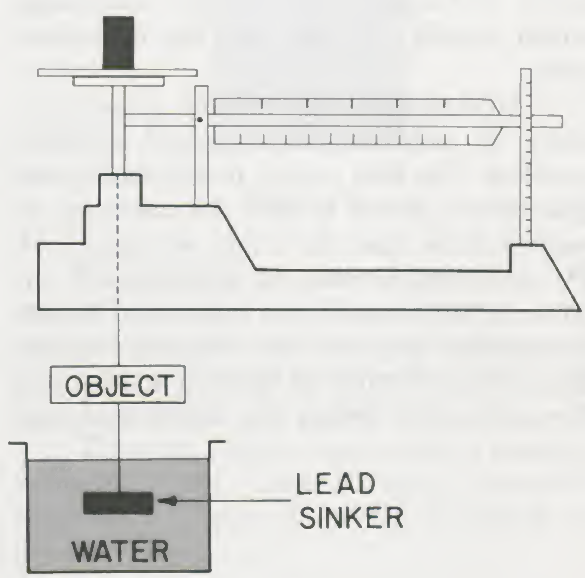


Figure 20.

4. Let the sinker pull the object down beneath the surface of the water. Be sure the sinker doesn't touch bottom. Now measure the apparent mass of the object and the sinker, with both in the water ($M_W + M_{SW}$).

The difference in these values is:

$$(M_A + M_{SW}) - (M_W + M_{SW}) = (M_A - M_W)$$

Thus, the volume is:

$$V = (M_A - M_W)/(1\ \mathrm{g/cm^3})$$

$$= [(M_A + M_{SW}) - (M_W + M_{SW})]/(1\ \mathrm{g/cm^3})$$

Finally, the density of the object is:

$$\rho_x = \frac{M_A}{V} = \frac{M_A\ (1\ \mathrm{g/cm^3})}{[(M_A + M_{SW}) - (M_W + M_{SW})]}$$

5. Calculate the densities of the objects which float.

The use of a gram balance to determine the volume of an object demonstrates two practical technical principles. First, there is often more than one way to make a satisfactory measurement. Second, different measuring techniques have different accuracies, which usually depend upon the instrument used.

Archimedes developed his principle in order to solve a very practical technical problem. The king wished to purchase a new gold crown. Before he paid the craftsman, he wanted to be sure the crown was pure gold. He asked Archimedes to determine if the crown was pure gold or a mixture of metals. Archimedes measured the buoyant force on the crown in water to provide the answer. Non-destructive testing is a widely used engineering technique. Surely, this is one of the earliest examples recorded.

QUESTIONS FOR EXPERIMENT B-2

1. A partially filled fish tank rests on a balance. A fish is put into the tank. The fish doesn't touch the sides. Does the balance indicate more weight? Explain your answer.

2. In Experiment B-2 we ignored the buoyant force of air on the objects. Show that this was a reasonable procedure to follow. (Density of air at standard conditions = .00129 g/cm^3)

3. Suggest a method other than using a hydrometer to measure the density of an unknown liquid. Write out a detailed procedure. Obtain a liquid sample and determine its density and specific gravity. Check your answer with a hydrometer. List the most important sources of error carefully.

4. A partially inflated rubber inner tube is just barely floating on the surface of a pond. It is pulled down 10 ft by a swimmer and released. Will it rise or sink? Explain.

5. A 1 ft^3 block of cement has an apparent weight of 330 lb when it is submerged in a lake. What is the buoyant force on it?

ATMOSPHERIC PRESSURE

You have seen that pressure in a liquid depends on both its density and the depth. The same is true for gases. We live at the bottom of an "ocean" of air, and the pressure of the atmosphere is significant. However, the density of the atmosphere varies with altitude, so Equation (7) can't be used to compute air pressure. Figure 21 shows that a column of air 1 ft^2 in cross-sectional area, and extending to the top of the atmosphere, weighs over a ton! The pressure at the bottom of this column can be calculated.

$$P\,(\text{atmosphere}) = P_A = \frac{2120\ \text{lb}}{1\ \text{ft}^2} = \frac{2120\ \text{lb}}{144\ \text{in}^2}$$

$$P_A = 14.7\ \text{lb/in}^2 \cong 15\ \text{lb/in}^2$$

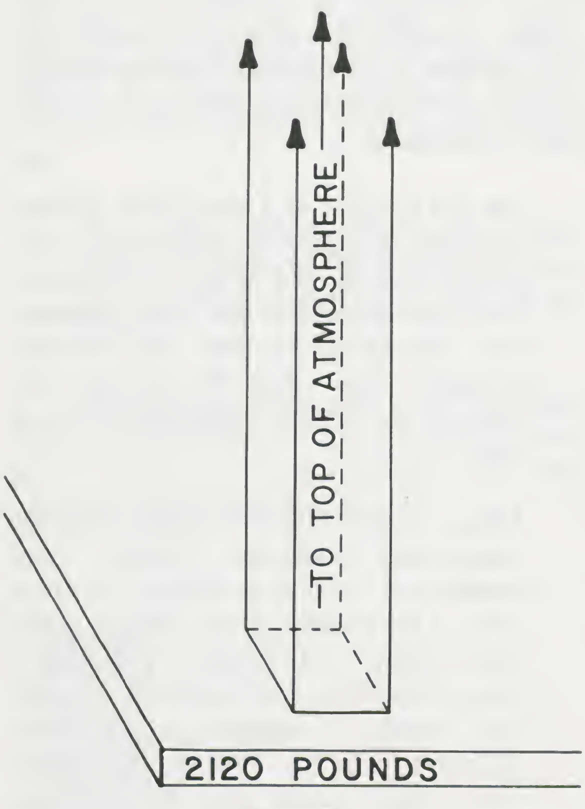


Figure 21.

You have probably encountered this value before. A column of water would have to be nearly 34 ft (10 m) high and a column of mercury would need to be 2½ ft (76 cm) high to exert the same pressure. As you know from weather forecasts, the actual pressure exerted by the atmosphere varies considerably. This pressure is a typical value at sea level and is called a *standard atmosphere.* Atmospheric pressure is often expressed in terms of the height of a column of liquid that produces the same pressure. A *mercury barometer,* like that in Figure 22, measures atmospheric pressure in terms of the height of a column of mercury which is balanced by air pressure.


Figure 22.

Since the air pressure decreases as one ascends above sea level, *aneroid barometers,* calibrated for altitude, are used as altimeters in airplanes. You will study both mercury and aneroid barometers in Section C.

There are a great many units of pressure currently in use. Table III gives a standard atmosphere (1 atm) in terms of several of them.

Table III.
A Standard Atmosphere in Several Systems of Units

$$\begin{aligned} 1\ \text{atm} &= 14.7\ \text{lb/in}^2 \\ &= 760\ \text{torr} \\ &\quad \text{(formerly millimeters of mercury)} \\ &= 29.9\ \text{inches of mercury} \\ &= 1.013 \times 10^6\ \text{dyn/cm}^2 \\ &= 1.013 \times 10^5\ \text{Pa}\ (1\ \text{Pa} = 1\ \text{N/m}^2) \end{aligned}$$

GAUGE PRESSURE

Does air pressure have anything to do with the pressure in liquids? The answer is yes and no. Atmospheric pressure acts on any exposed surface of a liquid. At a depth h below the liquid surface, the total pressure P is the sum of atmospheric pressure and the pressure due to the liquid. The pressure due to the liquid is Dh. The total pressure is then

$$P = Dh + P_A = hD + 14.7 \text{ lb/in}^2$$

This is usually called an *absolute pressure.* Usually liquid pressure problems require *only* the pressure due to the liquid. Such pressures are called *gauge pressures,* because most gauges read the pressure above atmospheric pressure. For example, a tire pressure gauge reads gauge pressure; inflating a tire "to 30 lb" means raising its pressure to 30 lb/in^2 above atmospheric pressure.

Note that in previous sections of the module we have been referring to gauge pressure (Dh) most of the time. How would the inclusion of atmospheric pressure affect the answers of the earlier examples?

The following example should help to clarify the difference between gauge pressure and absolute pressure.

Example 11. Suppose you have two pressure gauges, one of which reads absolute pressure and the other gauge pressure. What is the reading on each gauge at the surface of a lake and at a depth of 15 ft?

Solution. At the surface the absolute pressure is one atmosphere, or 14.7 lb/in^2. Therefore at the surface

$$P = P_A = 14.7 \text{ lb/in}^2$$

$$P_{\text{Gauge}} = 0$$

The gauge pressure (pressure due to the water) at a depth of 15 ft can readily be determined. Adding one atmosphere to it gives the absolute pressure. Thus at 15 ft

$$P_{\text{Gauge}} = Dh = (62.4 \text{ lb/ft}^3) \times (15 \text{ ft})$$

$$= 93.6 \text{ lb/ft}^2$$

$$= 6.5 \text{ lb/in}^2$$

Then

$$P = 14.7 \text{ lb/in}^2 + 6.5 \text{ lb/in}^2$$

$$= 21.2 \text{ lb/in}^2$$

EXPERIMENT B-3. A Closed Hydraulic System

In this experiment you will investigate the properties of a two-piston, closed hydraulic system, using two medical syringes which are coupled together. In a hydraulic system, matter is conserved (if there isn't a leak). Since a liquid is incompressible, the total volume of liquid in a closed system also remains constant. This may seem obvious, but if the system were pneumatic (air-operated), the enclosed volume *would* change, since air is easily compressed.

1. Use the syringes as a two-piston system, as shown in Figure 23. Demonstrate that matter is conserved as oil is transferred from one cylinder to the other. Measure the diameters of the two syringe plungers. Note that the syringes are marked for volume displacement in cc (cm^3).

2. Using a range of slotted weights and the two-syringe hydraulic system, hang weights up to 600 g so that the force acts on the smaller piston. Using a spring push balance to measure the output force, determine the output force after each weight is applied to the small-diameter cylinder. Record the output force, then gently tap the mounting board to break the frictional forces impeding the motion of the pistons. (This frictional force is termed the *breakaway force* in hydraulic technology.) Record the new value of the output force.

Figure 23. Two syringes coupled together to form a two-piston hydraulic system.

3. Compute the pressure at the face of each plunger.

4. Calculate the ratio of output force (after tapping) to input force for each weight.

CLOSED SYSTEMS–PASCAL'S LAW

While many characteristics of liquids can be explained by studying free-standing liquids in open containers, at least one important property relates to liquids in closed systems–systems which are not open to the atmosphere. Automobile brake systems and hydraulic jacks are examples of closed systems. The behavior of closed systems was studied by Pascal, an eighteenth century scientist. His conclusion is known as *Pascal's Law:*

> In a closed system filled with a fluid, pressure applied to the fluid in one part is transmitted equally to all parts of the system.

Frictional effects may have partially masked this effect in Experiment B-3, but your results for the pressures on the two plungers should have been close.

The hydraulic press and hydraulic jack are devices which work through the application of Pascal's Law. A discussion of these devices should lead to an understanding of this important principle of physics. Figure 24 shows a simplified version of the hydraulic jack. The basic parts are:

a. The enclosed fluid–usually oil.

b. A large piston which supports the load.

c. A small piston which applies the input force.

Let's start with the system in balance. (Pressure differences due to differences in height are small compared to the total pressure in the jack; therefore, we will ignore them.) The applied force on the small piston produces a pressure given by

$$P = \frac{F_1}{A_1}$$

Pascal's Law states that this same pressure will be transmitted to the large piston.

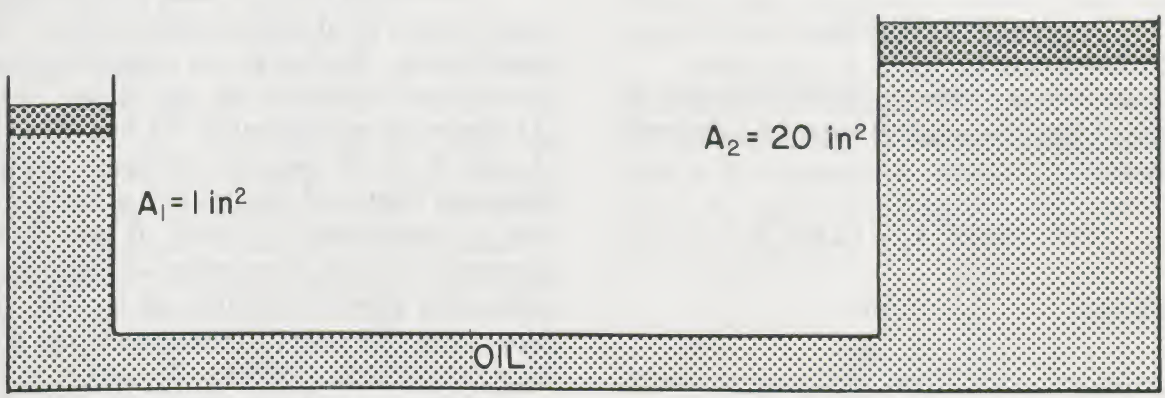


Figure 24. When the large piston moves up 1 in, the small piston must move down 20 in, and 1 lb applied to the small piston produces 20 lb of force on the large one.

$$P = \frac{F_2}{A_2}$$

Thus the force on the large piston is

$$F_2 = PA_2$$

$$= \frac{F_1}{A_1} A_2 = F_1 \frac{A_2}{A_1}$$

Or

$$\frac{F_2}{F_1} = \frac{A_2}{A_1} \tag{8}$$

For circular pistons, $A = (\frac{1}{4})\pi d^2$ and

$$\frac{F_2}{F_1} = \frac{\pi d_2^2/4}{\pi d_2^2/4}$$

Or

$$\frac{F_2}{F_1} = \frac{d_2^2}{d_1^2} \tag{9}$$

In other words, the force is multiplied by the ratio of the squares of the diameters. It sounds great to multiply force, but you may be sure there is a catch somewhere. You can't get something for nothing. Let's look at S_1, the distance the small piston must move in order to raise the large piston up a distance S_2. To move the large piston up S_2 requires a volume of oil given by

$$V = (\tfrac{1}{4})\pi d_2^2 S_2$$

Because the oil is incompressible, this must be equal to the volume of oil moved by the small piston:

$$V = (\tfrac{1}{4})\pi d_1^2 S_1 = (\tfrac{1}{4})\pi d_2^2 S_2$$

Therefore,

$$d_2^2 S_2 = d_1^2 S_1$$

Or

$$\frac{S_1}{S_2} = \frac{d_2^2}{d_1^2} = \frac{F_2}{F_1} \tag{10}$$

Thus, the smaller piston moves a longer distance than the larger one. In fact, in this ideal case the ratio of distances moved is just the ratio of output to input forces. For example, in the system of Figure 24 the small piston moves 20 times as far as the large piston and the force is multiplied 20 times. Rewriting Equation (10) shows that the product of force times distance is the same for the two pistons:

$$F_1 S_1 = F_2 S_2 \tag{11a}$$

The product of a force and the distance through which the force is applied is the physical definition of the term *work.* (This is a very special use of the term, quite different from our everyday meaning. A displacement parallel to the direction of the applied force must occur if physical work is to be done.) Thus, Equation (11a) means that work-in equals work-out ideally:

$$W_1 = W_2 \tag{11b}$$

In other words, the hydraulic jack doesn't reduce the amount of work one must do; it reduces the force needed but increases the distance through which it must be applied. If there is no friction (as we assumed in the above derivation), the work output equals the work input.

Experience tells you that hydraulic jacks don't raise a load with a single stroke of the small piston. Real jacks use repeated strokes of the small piston to do this lifting; Figure 25 shows this refinement of the jack. Sections A and B of the jack are the same as those shown in Figure 24. Section C is a reservoir of oil at atmospheric pressure. D and E are one-way valves. If the pressure on the right of valve D is greater than that on the left, the ball closes the valve. Conversely, if the pressure on the left is greater, the ball is forced out of the cup and the valve opens. Thus the

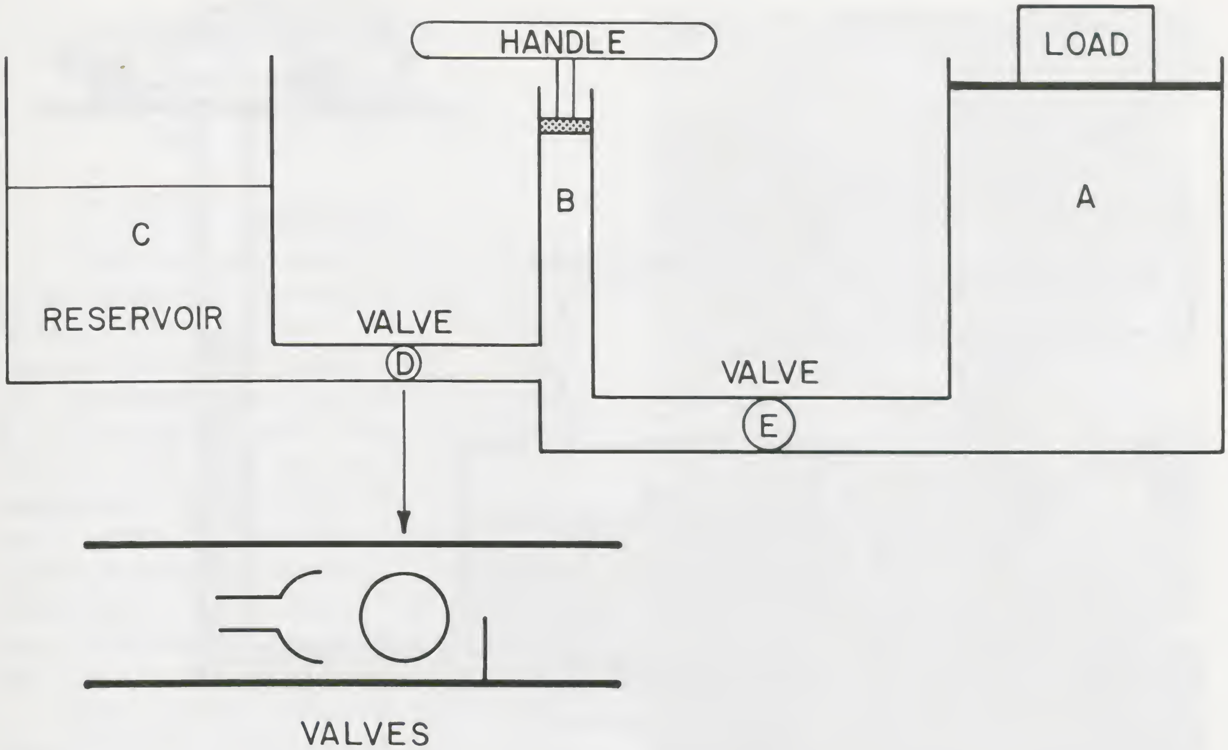


Figure 25. A hydraulic jack.

liquid will flow only from left to right through this valve. Valve E works exactly the same way.

When the piston in the small cylinder is pushed downward, valve D is forced shut and valve E is opened. This admits liquid to the large cylinder and causes the load to rise. If the small piston is raised, the valves reverse positions and liquid is drawn from the reservoir, refilling the cylinder at B. Add a lever (handle) to B and you have a hydraulic jack. (How do you get the load down?)

In garages and gas stations many hydraulic lifts employ a combination of a gas (air) and a liquid (oil). Multiple use is made of the air compressor, which also inflates tires, runs pneumatic tools, and helps to signal incoming business. In the automobile lift, the direct pressure from the small piston is replaced by air pressure from the compressor. For example, in an automobile lift, if the compressor output is 40 lb/in^2, and the lift piston is 12 in in diameter (113 in^2 area), the lift can support 4520 lb. See Figure 26.

FRICTION

We started the discussion of the hydraulic jack by neglecting friction; now let's put it back. With friction in the system, some work is lost and the output work W_2 no longer equals the input work W_1. We define the efficiency E:

$$E = \frac{W_2}{W_1} \times 100\%$$

This is the percentage of the work input which is converted into useful output.

Efficiency is often defined in terms of mechanical advantages. The *actual mechanical advantage* (*AMA*) includes frictional effects, and it is expressed in terms of the applied force F_1 and the actual output force 4 F_2 (load):

$$AMA = \frac{F_2}{F_1} \qquad (12)$$

This is just the ratio you computed in Experiment B-3.

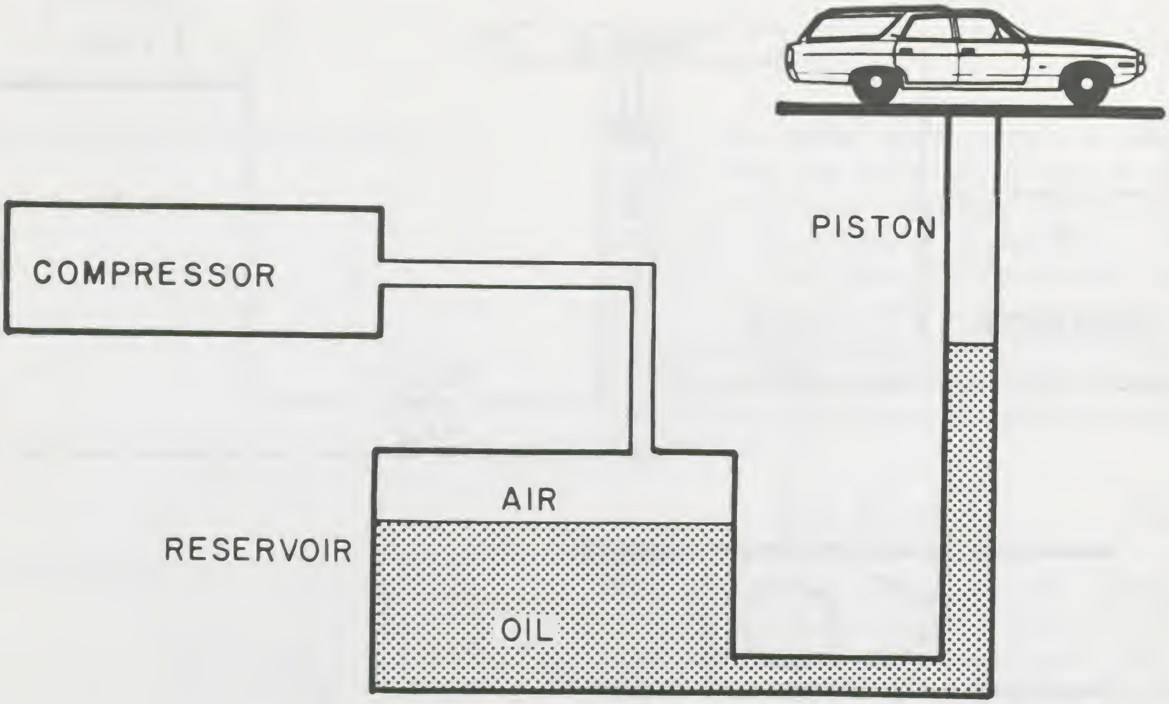


Figure 26.

The *ideal mechanical advantage* (*IMA*) ignores friction, and is usually expressed in terms of distances or dimensions. It is the value that F_2/F_1 would have in the absence of friction. Since Equation (10) was derived on the basis of no friction, we can use it to determine the ideal mechanical advantage for the jack:

$$IMA = \frac{F_2}{F_1}(\text{ideal}) = \frac{d_2^2}{d_1^2} = \frac{S_1}{S_2} \qquad (13)$$

Now the efficiency can be expressed in terms of mechanical advantages:

$$E = \frac{W_2}{W_1} \times 100\% = \frac{F_2 S_2}{F_1 S_1} \times 100\%$$

Rearranging this gives:

$$E = \frac{F_2/F_1}{S_1/S_2} \times 100\%$$

Or

$$E = \frac{AMA}{IMA} \times 100\%$$

Example 12. A hydraulic jack has a small-piston diameter of 1 in and a large-piston diameter of 9 in. It is found that, to lift a 2000-lb load, a force of 30 lb must be applied to the small piston. Determine

a. The ideal mechanical advantage.

b. The actual mechanical advantage.

c. The efficiency of the jack.

Solution.

a.

$$IMA = \frac{d_2^2}{d_1^2} = \frac{(9\text{ in})^2}{(1\text{ in})^2} = 81$$

b.

$$AMA = \frac{F_2}{F_1} = \frac{2000\text{ lb}}{30\text{ lb}} = 67$$

c. The efficiency is the ratio:

$$E = \frac{AMA}{IMA} \times 100\% \cong \frac{67}{81} \times 100\%$$

$$E \cong 82\%$$

If this jack were operated with a levered handle which has a mechanical advantage of 10 (lever arms in the ratio 10:1), the applied force on the end of the handle would be only 3 lb. (Mechanical pivots are almost frictionless if good bearings are used.) Even if the friction were so high that the efficiency of the mechanical lever was only 50%, a 6-lb force input would produce the required 30 lb needed to operate the hydraulic lift. With this combination of mechanical and hydraulic force multiplication, even a small child could lift a 1-ton load.

Power steering units for automobiles, large hydraulic hoists, cherry-picker cranes, and dump-truck hoists are essentially like a hydraulic jack in which the lever and small piston are replaced by a pump which forces liquid into the large cylinder. Technologists have certainly put Pascal's Law to good use. One example is shown in Figure 27.

Figure 27. Hydraulic systems do much of the heavy work in our technological world.

SUMMARY

Pressure is the ratio of the normal force to the area over which the force acts and is usually expressed in Pa (N/m^2), lb/in^2, or dyn/cm^2.

The pressure at any point a distance h below the surface of a liquid is equal to the product of weight density (D) and depth (h).

$$P = Dh = \rho gh$$

Archimedes' Principle states that, when an object is placed in a liquid, it is buoyed up by a force equal to the weight of the displaced liquid.

The absolute pressure is equal to the gauge pressure plus the atmospheric pressure.

$$P = P_G + P_A$$

Pascal's Law states that the pressure applied to an enclosed system is transmitted equally to all parts of the system.

The hydraulic jack and the hydraulic press both operate according to Pascal's Law. The actual mechanical advantage of these devices is:

$$AMA = \frac{F_2}{F_1} = \frac{\text{Output force}}{\text{Input force}}$$

The ideal mechanical advantage of these devices is:

$$IMA = \frac{d_2^2}{d_1^2} \text{ where } d = \text{piston diameter}$$

The efficiency is:

$$E = \frac{\text{Work out}}{\text{Work in}} \times 100\% = \frac{AMA}{IMA} \times 100\%$$

PROBLEMS

1. A submarine's hull can withstand a water pressure of 20 ton/ft^2. What is its safe diving limit? (The weight density of sea water is about 64 lb/ft^3.)

2. An ice cube floats in a glass brim full of water. When the ice cube melts, does the glass overflow? State a precise reason for your answer. Perform a simple experiment to prove your results.

3. Gin has a density of about 0.90 (depending on the proof rating). Will an ice cube float higher or lower in a glass of gin than in a glass of water? Will the ice cube float in pure alcohol (200 proof)? Test your answers on a few friends.

4. A student reading a physics text is floating on a raft in her swimming pool. In frustration she throws her text into the pool from the raft. Does the water level rise in the pool? Explain your answer.

5. Using your data from Experiment A-1, calculate the actual mechanical advantage of the automotive jack.

6. The small piston of a hydraulic jack has a cross-sectional area of 2 in^2. The large piston has cross-sectional area of 40 in^2. What is the ideal mechanical advantage of this jack? How many 4-in strokes of the small piston are required to raise the load on the large cylinder 100 in?

7. The surface of a vinyl floor covering has a breaking strength of 250 lb/in^2. Will a 105-lb girl wearing shoes having heels with an area of 0.35 in^2 break the surface? State clearly any assumptions that you make. (In the days of "spike" heels, this was a serious problem in many public buildings.)

8. A block of aluminum 2 in × 6 in × 1 in floats on its large face in a pool of mercury.

   a. What pressure does the weight of the aluminum exert on the mercury?

   b. What is the buoyant force on the aluminum?

c. How much mercury is displaced by the aluminum?

9. A hydraulic cylinder whose diameter is 2.5 cm is coupled to a cylinder of diameter 12.5 cm. What force would be required at the smaller cylinder to provide an output force of 1100 lb if the system operates with 88% efficiency?

10. A window on a spacecraft has a diameter of 12 in. The spacecraft is pressurized at 0.8 atm. What pressure must the window glass withstand in lb/in^2? What total force is exerted on the window?

11. The face-mask window of a skin diver has an area of $100 \ in^2$. The pressure inside is 1 atm. At what depth will the water pressure be ten times the inside pressure? What will the total force be on the outside of the window?

12. From your answers to questions 10 and 11, can you explain why it is easier to go into outer space than to go to the bottom of the ocean (46,000 ft)?

13. A naval depth charge is set to go off when its pressure sensor detects $150 \ lb/in^2$ pressure. If the density of sea water is $64 \ lb/ft^3$, at what depth will the charge explode?

14. A Polaris submarine crew wishes to launch a missile from a depth of 80 ft. To what pressure must the missile firing tube be pressurized to open the protective door?

15. An Eskimo tries to sneak up on a seal by stepping onto a circular ice floe 3 ft in diameter and 18 in thick. The floe is in salt water ($D = 64 \ lb/ft^3$). If the Eskimo weighs 140 lb, and the specific gravity of salt ice is 0.90, will the ice floe support him?

16. A 23-lb bicycle has a 137-lb rider on it. The weight is evenly distributed to both wheels. If the pressure in the tires is $80 \ lb/in^2$, how much tire area contacts the road?

17. As the eye of a tornado passes over the roof of a house, the pressure outside the roof suddenly drops to 27 in of mercury. If the windows and doors are all closed, the inside pressure does not change rapidly. Calculate the pressure differential in lb/in^2 if the inside pressure remains at 30 in of mercury. If the house is 30 ft $\times$ 50 ft, calculate the total force lifting the roof as the tornado passes. What should one do with the windows when one hears a tornado warning for the area?

18. A boy weighs 185 lb. When he dives into a pool, he displaces 22 gal of water. His 110-lb girlfriend displaces 13 gal. Whose body is more dense?

SECTION C-1

FLUIDS IN MOTION: HYDRODYNAMICS

The study of the behavior of fluids in motion is called *hydrodynamics.* Fluids in motion display properties that differ dramatically from fluids at rest.

One of these properties is *frictional drag.* Real liquids such as water, beer, and oil experience a frictional force, called *drag,* as they flow through pipes or over surfaces. The result is that the force exerted by a moving liquid on the walls of the pipe has a component parallel to the pipe. Static fluids always exert forces perpendicular (normal) to the walls of the container. The frictional effect causes a loss of mechanical energy and a loss of pressure as the fluid flows through the pipe. This effect is easily seen in common garden hoses. The longer the hose, the less the pressure at the end of the hose.

A detailed study of friction is beyond the scope of this module, and the following discussions assume that the effect can be ignored. Frictional effects only cause a change in numerical values; they do not change the main features of the fluid behavior.

A second difference between static fluids and flowing fluids is illustrated by Experiment C-1. Differences in the velocity of a fluid produce differences in pressure in the fluid. After the experiment, an optional subsection explains why this pressure difference exists, and derives an equation which is used to compute the difference.

EXPERIMENT C-1. The Bernoulli Effect

In this experiment you will see several examples of the way in which the motion of a fluid, air, affects its pressure. You will not be required to record data in the usual way, but you should write down what you see in each case. Sketches may be helpful.

Procedure

1. Hold a piece of notebook paper in front of your mouth as in Figure 28. What happens to the paper when you blow across its upper surface?

Figure 28.

2. Stick a pin through the center of a 3 in X 5 in index card. Insert the pin into the center hole of an ordinary household spool, so that the card covers the end of the spool. Blow through the other end of the spool. What happens?

3. Fold a 3 in X 5 in index card as shown in Figure 29. Place the card on the edge of a table and blow through the "tunnel" formed by the card and the table top. What happens? Can you blow the card off the table in this way?

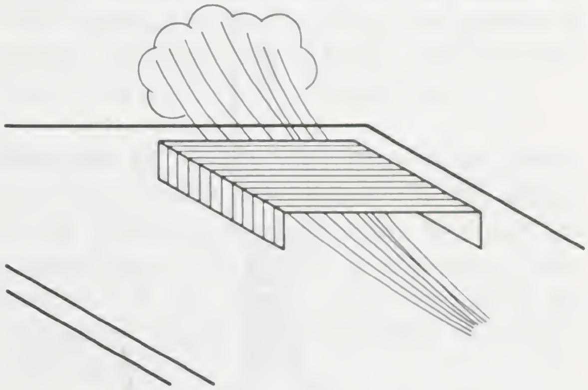


Figure 29.

4. Attach an air supply hose to the nozzle provided. Turn on the air and place a Ping-Pong ball in the air flow as in Figure 30. Describe what happens as the air stream is slowly tilted away from the vertical position.

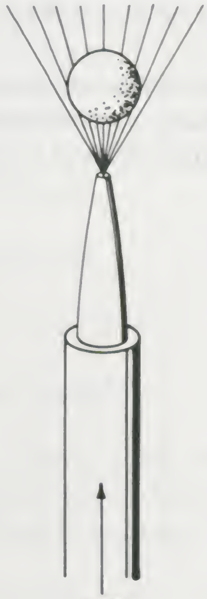


Figure 30.

5. Replace the nozzle with a funnel. Turn the air on and place a Ping-Pong ball in the throat of the inverted funnel (see Figure 31). Release the ball and describe what happens.

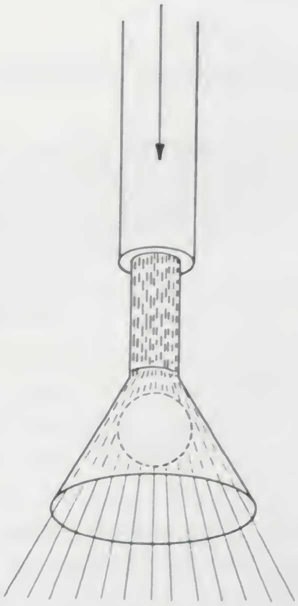


Figure 31.

6. Tape a piece of thread to a Ping-Pong ball and suspend it near a stream of water from a faucet. What happens?

7. Tape some thread to a second Ping-Pong ball and hang the two balls so that they are the same distance below a horizontal support and about 5 cm apart. Blow gently between them. What happens? Describe the result of blowing harder.

8. Fill the U-shaped glass tube about half full of water. Blow across one open end of the tube and ask a classmate to describe the results. Be careful to blow horizontally and not down into the tube.

9. Observe the behavior of the atomizer when you squeeze the bulb. Do you see any connection between the behavior of this device and the effects observed in steps 1 through 8?

10. Examine the aspirator carefully. Then hook up the aspirator supply hose to a water outlet. When you are sure that the drain hose is properly placed, turn on the water and hold your finger over the open arm of the "T." What happens?

Think about the observations you have just completed. Can you identify any common element of these effects? If so, you may be able to guess the physical reason for the behavior of each of the nine "devices."

BERNOULLI'S PRINCIPLE

Did you guess that the pressure exerted by a fluid is affected by the motion of that fluid? The greater the fluid speed, the less pressure it exerts. This important physical fact is called *Bernoulli's principle.*

The Bernoulli principle can be derived by applying the principle of energy conservation to a small volume of the fluid as it moves. Such an analysis, which we do later in an optional subsection, leads to a general equation which is valid for incompressible fluids. Referring to Figure 32, the Bernoulli equation is

$$P_1 + \rho g h_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \rho g h_2 + \tfrac{1}{2}\rho v_2^2 \qquad (14)$$

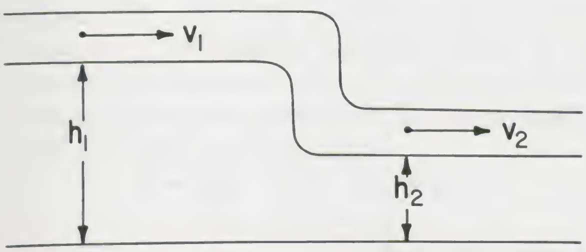


Figure 32.

where:

P_1, P_2 = absolute pressure in the pipe at points 1 and 2.

ρ = density of the fluid.

$g = 9.8\ \mathrm{m/s^2}$ = acceleration due to gravity.

h_1, h_2 = heights of pipe, measured from some arbitrary level.

v_1, v_2 = fluid speeds at points 1 and 2.

In many cases of practical interest, the two sections of the "stream" are at equal height ($h_1 = h_2$) and the Bernoulli equation reduces to

$$P_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \tfrac{1}{2}\rho v_2^2 \qquad (15)$$

Note that this agrees with what you observed in your experiments: the point at which the fluid speed is *greater* is where the pressure is *lower.* The following example illustrates the use of this equation in a real device.

Example 13. Figure 33 illustrates an aspirator. Suppose the water speed into the aspirator is 6.66 m/s and water speed just past the constriction is 20.5 m/s. If the input pressure is $2 \times 10^5\ \mathrm{N/m^2}$ (1.97 atm), find the "pumping" pressure. (The density of water is $1000\ \mathrm{kg/m^3}$.)

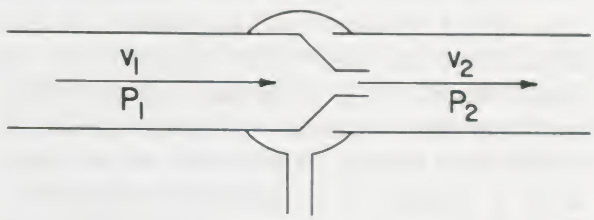


Figure 33.

Solution. We can apply Bernoulli's equation to the points 1 and 2 before and after the constriction. The pumping pressure P_2 is the pressure following the constriction. We know the following quantities:

$$v_1 = 6.66\ \mathrm{m/s}$$

$$v_2 = 20.5\ \mathrm{m/s}$$

$$P_1 = 2 \times 10^5\ \mathrm{N/m^2}$$

$$\rho = 10^3\ \mathrm{kg/m^3}$$

Since the heights in the two parts of the aspirator are the same, we can use the simpler form of the Bernoulli equation:

$$P_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \tfrac{1}{2}\rho v_2^2$$

Or:

$$P_2 = P_1 + \tfrac{1}{2}\rho v_1^2 - \tfrac{1}{2}\rho v_2^2$$

$$= 2 \times 10^5\ \mathrm{N/m^2} + \tfrac{1}{2}(10^3\ \mathrm{kg/m^3}) \times$$

$$(6.66\ \mathrm{m/s})^2 - \tfrac{1}{2}(10^3\ \mathrm{kg/m^3})(20.5\ \mathrm{m/s})^2$$

$$= (2 \times 10^5 + .22 \times 10^5 -$$

$$2.10 \times 10^5)\ N/m^2$$

$$= .12 \times 10^5\ N/m^2$$

DERIVATION OF THE BERNOULLI EFFECT (OPTIONAL)

Figure 34 shows a tank of water with an outlet at the bottom. Water leaves the bottom at a speed v_2 and at atmospheric pressure P_A. As you know, the greater the depth of the water, the faster it flows from the outlet. We seek first a mathematical expression for the dependence of the water velocity on its depth. Such an expression can be obtained by applying the principle of conservation of mechanical energy, but to do so we must make a number of simplifying assumptions. For example, we shall assume that there is no friction and that the flow is perfectly smooth (*laminar*).

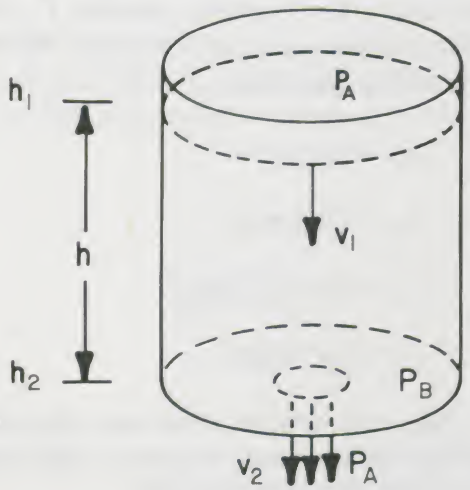


Figure 34.

Another idealized assumption we make is that v_1 is the velocity of the water flow everywhere in the main part of the container. Then a mass M of water in the container has a kinetic energy $\frac{1}{2} Mv_1^2$. If that same mass leaves the container, its kinetic energy as it leaves is $\frac{1}{2} Mv_2^2$. Thus there has been an increase in the kinetic energy of the mass M given by

$$\Delta KE = \tfrac{1}{2} Mv_2^2 - \tfrac{1}{2} Mv_1^2 \qquad (16)$$

We assume that energy is conserved. Therefore, this increase in kinetic energy must have come from somewhere. Where? The level of water in the container drops somewhat, thereby decreasing the gravitational potential energy available. The gain in kinetic energy must be equal to this loss in potential energy. The decrease in height is exactly what would occur if the mass M were removed from the top of the container and placed at the bottom. Therefore, the change in gravitational potential energy is negative and is given by:

$$\Delta PE = Mg\,(h_2 - h_1) = -Mgh \qquad (17)$$

The law of conservation of energy states that the total energy remains constant or, in other words, that the change in total energy is zero:

$$\Delta KE + \Delta PE = 0$$

Thus:

$$\tfrac{1}{2} M\,(v_2^2 - v_1^2) - Mgh = 0$$

Dividing by $\frac{1}{2} M$:

$$v_2^2 - v_1^2 = 2\,gh \qquad (18)$$

This tells us how fast the fluid leaves the container, expressed in terms of the depth h. A somewhat similar situation is discussed in the following example.

Example 14. Water falls from the top of a power dam to the river below, a drop of 80 m (260 ft). What kinetic energy does each kg of water have as it reaches the river bed? What velocity does the water have?

Solution. The kinetic energy gained just equals the potential energy lost as water falls 80 m.

$$\Delta KE = Mgh$$

$$= (1\ kg)\,(9.8\ m/s^2)\,(80\ m)$$

$$= 784\ J$$

Assuming that $v_1 = 0$ (the water has no kinetic energy as it dribbles over the top of the dam):

$$\Delta KE = \tfrac{1}{2} M v_2^2 = 784 \text{ J}$$

$$v_2^2 = 1568 \text{ m}^2/\text{s}^2$$

$$v_2 \approx 40 \text{ m/s}$$

The process whereby falling water converts gravitational potential energy to kinetic energy is the basis of hydroelectric power generation. The falling water collides with the blades of a turbine, converting the water's kinetic energy to rotational energy which drives the generators. The same principle has powered waterwheels for centuries.

Returning to the container of Figure 34, there is another way of looking at the situation. Certainly, it is the pressure which causes the water to be pushed out of the bottom. The surface of the water is at atmospheric pressure P_A. The difference between this and the pressure in the water at the bottom of the container P_B is

$$P_B - P_A = \rho g h$$

Combining this with Equation (18) gives:

$$v_2^2 - v_1^2 = (2/\rho)(P_B - P_A)$$

Notice that P_A, the atmospheric pressure, is also the pressure in the region where the water is flowing out of the bottom of the container. Now let $P_A = P_2$ and $P_B = P_1$, so that P_1 is the pressure in the region where the velocity is v_1 and P_2 is the pressure in the region where the velocity is v_2. We can now write the preceding equation as:

$$\rho/2\,(v_2^2 - v_1^2) = (P_1 - P_2) \qquad (19)$$

Now we have a way of calculating the velocity in terms of the pressure difference, or vice versa. It doesn't matter what causes the pressure difference. Equation (19) is true for any case of a liquid in continuous flow without friction. The pressure P_1 does not have to be due to the weight of a column of liquid, nor does P_2 have to be atmospheric pressure. Although Equation (19) was derived for the situation shown in Figure 34, it is also true for the situation shown in Figure 35, where three different regions are involved.

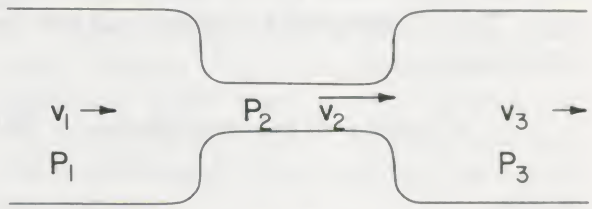


Figure 35.

An interesting conclusion can be reached by inspecting Equation (19): the pressure is smaller where the velocity is greater. This is exactly the Bernoulli effect that you observed in Experiment C-1.

We now consider the case in which the pipe carrying the fluid is not all at the same height, as shown in Figure 36. In this situation, there is a pressure difference due to the height difference, in addition to that due to the differences in velocities. We can add the *dynamic* pressure difference (due to a velocity difference) to the *static* pressure difference (due to a height difference) to get the *net* pressure difference.

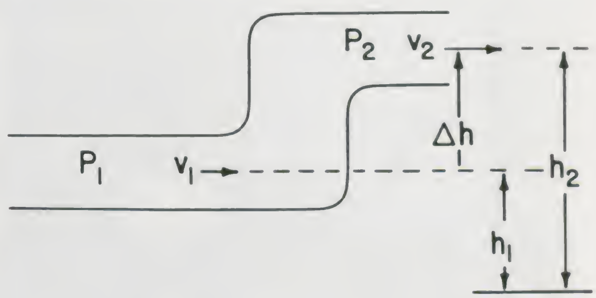


Figure 36.

$$P_1 - P_2 = (\rho/2)(v_2^2 - v_1^2) + \rho g (h_2 - h_1)$$

Note: if the pipe diameter does not change, v_2 must equal v_1. Why? We can rearrange the terms in this equation to get the *Bernoulli equation*:

$$P_1 + \rho g h_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \rho g h_2 + \tfrac{1}{2}\rho v_2^2 \tag{20}$$

Since the subscripts refer to any two points in the pipe, the sum of the three terms has the same value for all points along the pipe. That is, Bernoulli's equation can also be written as:

$$P + \rho g h + \tfrac{1}{2}\rho v^2 = \text{constant} \tag{21}$$

where:

P = absolute pressure (due to some impressed force)

$\rho g h$ = static pressure due to a depth h of liquid

$\tfrac{1}{2}\rho v^2$ = dynamic pressure due to fluid velocity v

The Bernoulli equation is valid only if certain conditions are met. We have already assumed that the fluid is incompressible and that there is no friction between the fluid and the walls of the pipes. In addition, we must assume that the flow is perfectly smooth, or *laminar*. *Turbulent* flow, involving *eddies* or *vortices,* is not adequately described by the Bernoulli equation.

Except for rather low velocity flow, the Bernoulli equation does not work well for gases, which are highly compressible. However, as you saw in Experiment C-1, the basic fact of increased velocity resulting in decreased pressure still holds.

OTHER HYDRAULIC DEVICES

The Aspirator

Consider a tube with a constricted throat, as in Figure 37. The constricted throat is called a *venturi*. If P_1 is comparatively large and the venturi has a very small diameter, the velocity V_2 can be quite large. Therefore P_2 is less than P_1 or P_3. In fact, P_2 can easily become *less* than the atmospheric pressure outside this system, P_A.

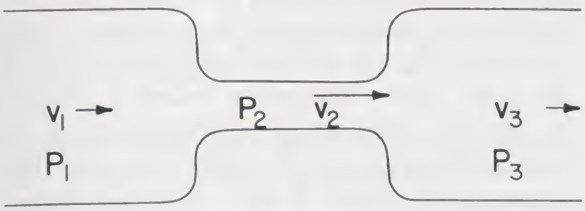


Figure 37. A venturi tube.

If we now connect a tube to the constricted throat, gas or liquid is drawn into the main stream because of the pressure difference (P_A - P_2) (Figure 38). Such a device is called an *aspirator*. Chemistry students often use an aspirator connected to an inverted funnel to exhaust the noxious fumes they frequently generate in the laboratory. If a gas, such as air, is forced through a venturi, the pressure drop at the constriction can be used to draw liquids into the gas stream. Atomizers, paint sprayers, aerosol cans, and garden sprayers thus are also aspirators. The carburator throat in your car is a venturi which draws gasoline from the bowl while mixing it with air.

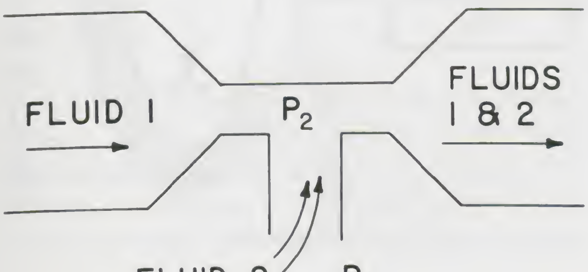


Figure 38. Aspirator.

Figure 39A shows a typical aspirator pump. (Figure 39B is a photograph of an aspirator.) Water flows in the top and out the bottom (into a sink). The horizontal arm can be connected to any container one wishes to pump.

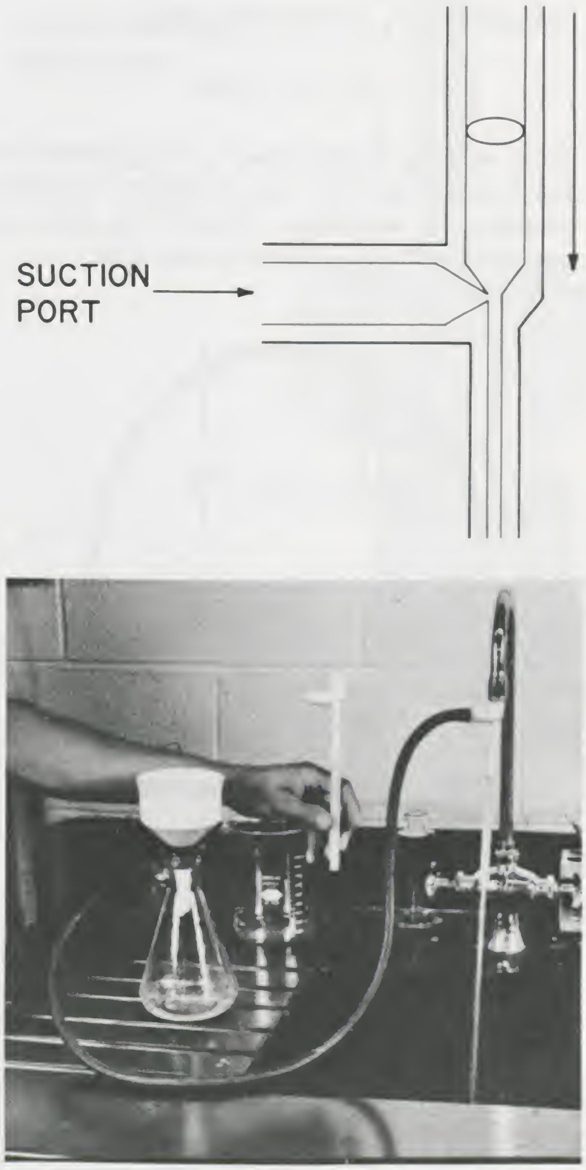


Figure 39A and B. A schematic and a photo of an aspirator.

The Siphon

A simple but useful hydraulic device is the *siphon,* which is simply a tube or hose connecting two containers of liquid, as shown in Figure 40. If the tube is filled with fluid, the siphoning effect causes fluid to flow from the higher container to the lower container.

To understand why the effect occurs, first imagine that a closed valve is inserted at the top of the hose (point C in Figure 40). The pressure P_1 on the left side of the valve is

$$P_1 = P_A - Dh_1$$

The pressure P_2 on the right side is

$$P_2 = P_A - Dh_2$$

Thus, with h_2 greater than h_1, the pressure on the left side of the valve is greater than the pressure on the right. Then if the valve is opened, the fluid will flow to the right.

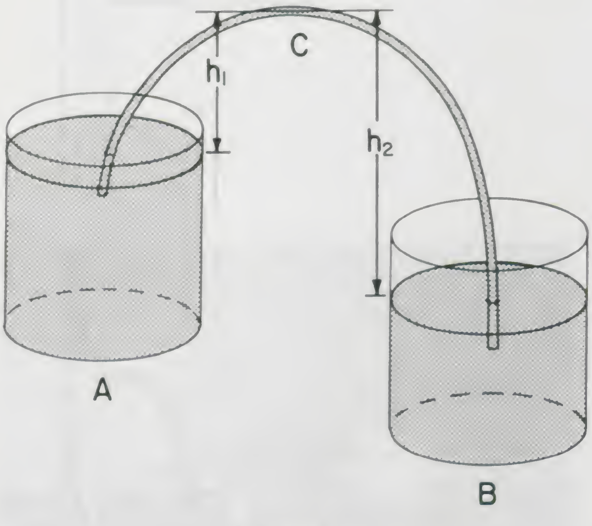


Figure 40.

You will find a problem if you try to siphon water over a wall more than 34 ft high; it won't work. Atmospheric pressure can support a column of water 34 ft high, but no higher. Thus, if the wall is higher than that, an attempt to siphon over it would fail. If one started with the hose full of water, the level would merely drop until its height was 34 ft in each arm, with a vacuum ($P \cong 0$) in the middle section. Then each arm would act as a water barometer. The system would sit there with nothing happening.

If you have never used a siphon, you would enjoy trying one out.

PRESSURE MEASURING DEVICES (GAUGES)

The Open-Tube Manometer

Perhaps the least complicated device used to measure static pressure is an *open-tube manometer,* shown in Figure 41. The fluid whose pressure is to be measured exerts a pressure P_M on one surface of the liquid in the tube. The atmosphere exerts a pressure P_A on the other surface of the liquid. If the two surfaces are at the same level, the two pressures are equal. If the two surfaces are a vertical distance h apart, the difference in pressure between the two arms is just that due to the column of liquid of height h. That is, $P_M - P_A = \rho g h$, where ρ is the density of the liquid in the tube.

If this result is not obvious, perhaps the following discussion will help. The pressure at

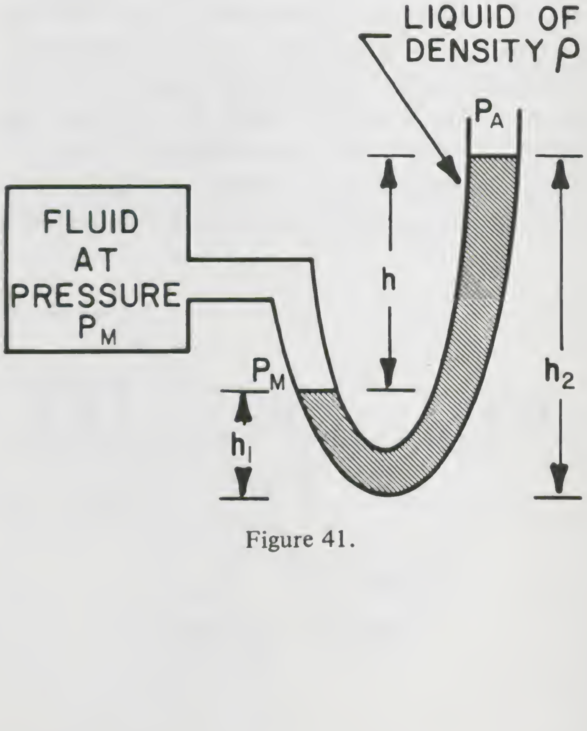


Figure 41.

the bottom of the left part of the tube is $P_M + \rho g h_1$ and the pressure at the bottom of the right side of the tube is $P_A + \rho g h_2$. At the bottom of the manometer, these two pressures are equal. Why? Then

$$P_M + \rho g h_1 = P_A + \rho g h_2$$

$$P_M - P_A = \rho g (h_2 - h_1) = \rho g h \quad (22)$$

The gauge pressure on the manometer is just $P_M - P_A = \rho g h$. The manometer scale may be calibrated in any convenient units.

Example 15. The fluid in an open-tube manometer is mercury ($\rho = 13.6\ \mathrm{g/cm^3}$). When it is attached to a cylinder of gas, the "pressure head" h produced is 10 cm. Determine the gauge pressure and the absolute pressure in $\mathrm{N/m^2}$.

Solution. The gauge pressure is the pressure due to the 0.1-m head of mercury.

$$P_G = \rho g h$$

$$= (13.6 \times 10^3\ \mathrm{kg/m^3})(9.8\ \mathrm{m/s^2})(0.1\ \mathrm{m})$$

$$= 1.34 \times 10^4\ \mathrm{N/m^2}$$

The absolute pressure is greater than the gauge pressure by one atmosphere.

$$P_M = P_G + P_A = P_G + 1\ \mathrm{atm}$$

$$= P_G + 1.01 \times 10^5\ \mathrm{N/m^2}$$

$$= (1.34 \times 10^4 + 1.01 \times 10^5)\ \mathrm{N/m^2}$$

$$= 1.14 \times 10^5\ \mathrm{N/m^2}$$

Barometers

The *barometer,* which was mentioned earlier, is an example of a *closed-tube manometer*. (See Figure 42.) The weight of a column of mercury is balanced by the force exerted on the open mercury surface by the

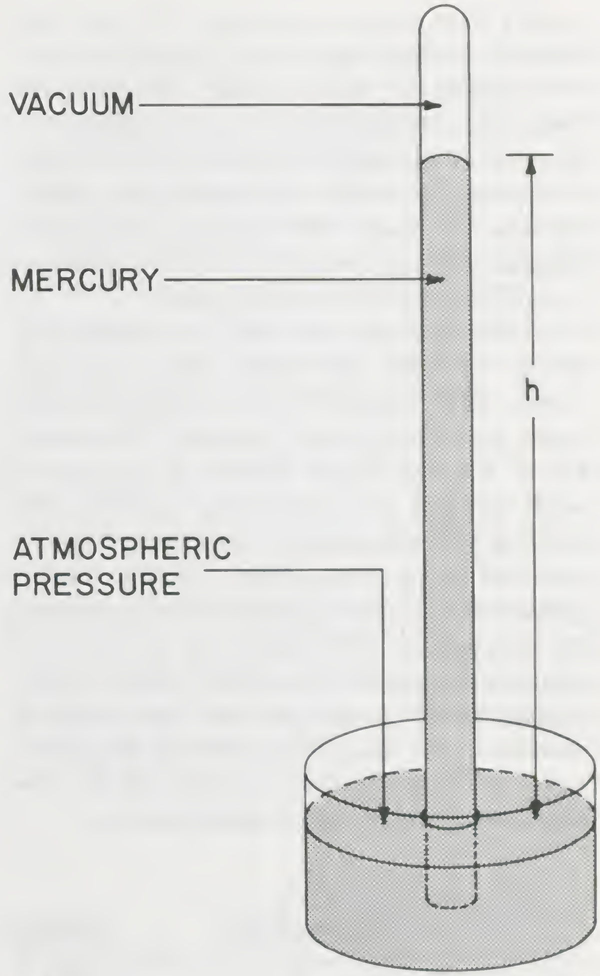


Figure 42.

pressure of the atmosphere. The space at the closed end of the tube exerts essentially zero pressure on the mercury column, so $P_A = \rho g h$. At one atmosphere of pressure, the height of the mercury column is

$$h = \frac{P_A}{\rho g} = \frac{1.01 \times 10^5\ \mathrm{N/m^2}}{1.36 \times 10^4\ \mathrm{kg/m^3} \times 9.8\ \mathrm{m/s^2}}$$

$$= .760\ \mathrm{m} = 760\ \mathrm{mm}$$

The height of the column of mercury in a barometer in millimeters of mercury or inches of mercury is most often used to

specify atmospheric pressure. You may have heard a weather report saying that the barometer reading is, for example, 29.92 in. This means that the measured value of atmospheric pressure at the weather station is 29.92 inches of mercury. Actually the measurement probably was *not* made with a mercury barometer. Instead, a more compact instrument called an *aneroid barometer* was likely used.

The aneroid barometer is another hydraulic device. The small metal can (see Figure 43) contains air at a pressure much lower than atmospheric pressure. The flexible lid of the can bends slightly in response to small changes in atmospheric pressure. This bending is measured by a mechanical system attached to a scale. The scale is usually calibrated in inches of mercury by comparing the response of the instrument to that of a mercury barometer. Since the value of atmospheric pressure decreases as one's altitude increases, the scale of an aneroid barometer can also be calibrated to read altitude. The instrument is then called an *altimeter*.

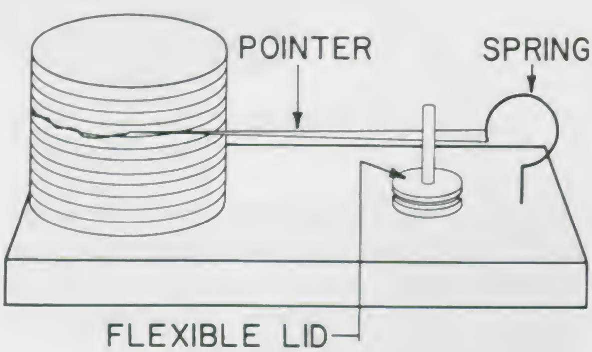


Figure 43.

The Bourdon Gauge

Manometers are awkward to use, so another type of gauge called a *Bourdon gauge* is frequently used in technical applications. The Bourdon-type gauge (Figure 44) is basically a curved tube sealed at one end. Pressure exerted through the open end of the tube tends to make the tube, which is fastened at its open end, straighten out. The movement of the free end causes a pointer to move over a scale. A familiar party favor, Figure 45, helps to illustrate the principle.

Figure 44.

Figure 45.

PRESSURE MEASUREMENTS

For moving liquids, we may measure any of three different pressures. One is the static pressure which would exist if the liquid were not moving. A second is the dynamic pressure, given by $\frac{1}{2} \rho v^2$. The third is the total pressure which includes both the static and dynamic pressures.

A Bourdon gauge or a manometer can be used to measure either static pressure or total pressure. Which pressure is measured depends on the way the instrument is connected to the pipe.

It is possible (see Figure 46A) to connect a gauge to the pipe in such a way that its presence does not significantly disturb the fluid flow. If the gauge is a manometer, the pressure difference across the two arms of the manometer is

$$P_A - P_1 = \rho_M g h$$

where ρ_M is the density of the liquid in the manometer, P_1 is the pressure in the pipe, and P_A is atmospheric pressure. Thus the pressure measured by the gauge is the difference between the atmospheric pressure and the pressure in the flowing fluid. The pressure in the fluid is then:

$$P_1 = P_A - \rho_M g h$$

or it is the difference between atmospheric pressure and the pressure measured by the gauge.

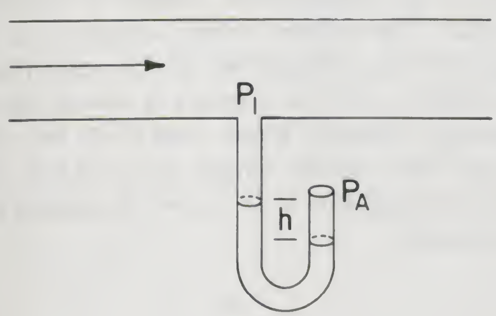


Figure 46A.

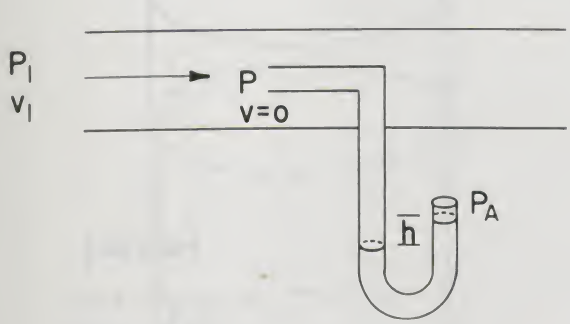


Figure 46B.

Alternatively, one can mount the gauge as shown in Figure 46B. The open probe facing into the flow is called a *Pitot tube* (pronounced pea-tow). The pressure in the tube increases until it offsets the pressure at the opening. In such a case, a "dead" point of no fluid flow is formed at the opening of the tube. One can apply Bernoulli's equation to the fluid to find the pressure difference between the opening of the tube and a point well removed from the probe opening

$$P_1 + \tfrac{1}{2}\rho_L v_L^2 = P + 0 \qquad (23)$$

where P_1 is the pressure in the flowing fluid, v_L is the fluid velocity, ρ_L is the fluid density, and P is the pressure at the opening of the tube. As above, the manometer displays the difference between atmospheric pressure and the pressure at the opening at the Pitot tube. That is

$$P_A - P = \rho_M g h \qquad (24)$$

$$P = P_A - \rho_M g h = P_1 + \tfrac{1}{2}\rho_L v_L^2$$

P is called the *total* pressure.

The Pitot Tube

Equation (23) suggests a way in which the velocity of a liquid can be measured. By using the two gauges of Figure 46, P_1 and P can be measured. Then the velocity-dependent part of the pressure (*dynamic pressure*) is the difference of the two readings:

$$\tfrac{1}{2}\rho_L v_L^2 = P - P_1 = P_V$$

$$v_L = \sqrt{2 P_V/\rho_L} \qquad (25)$$

Example 16. An arrangement such as that in Figure 46 is used to measure velocity of water in a pipe. If the difference between the two pressures is 2 lb/in^2, what is the velocity of the water?

Solution. We may apply Equation 25 using $D = \rho g$:

$$v = \sqrt{\frac{2\,gP_V}{D}}$$

$$= \sqrt{\frac{2\,(32\text{ ft/s}^2)\,(2\text{ lb/in}^2)}{62.4\text{ lb/ft}^3}}$$

$$= \sqrt{\frac{2\,(32\text{ ft/s}^2)\,(288\text{ lb/ft}^2)}{62.4\text{ lb/ft}^3}}$$

$$= 17\text{ ft/s}$$

A different device, also called a *Pitot tube,* combines the two measurements of Figure 46 into a single manometer. Thus it gives a direct reading of the pressure difference for moving and stationary parts of the fluid. As shown in Figure 47, the pressure in side A of the manometer is the pressure of the stationary fluid at the inner tube. The pressure in side B is the pressure of the fluid moving past the side holes in the Pitot tube.

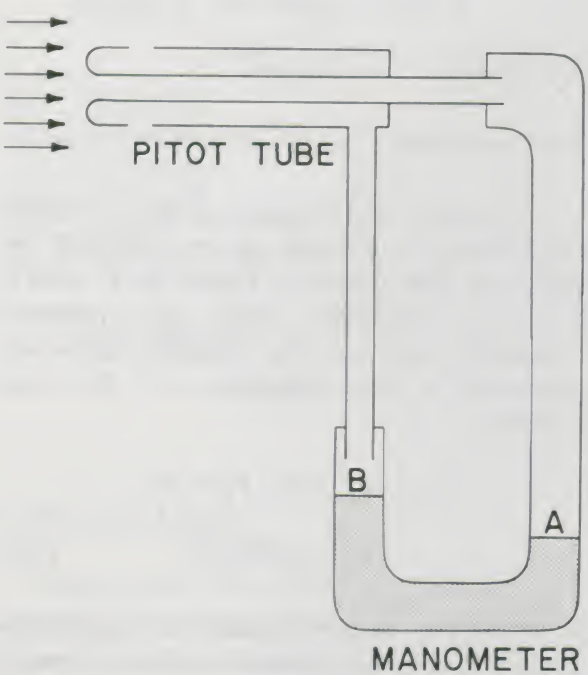


Figure 47.

The Pitot tube works well for measuring the velocity of gases, provided that the velocity is not so high that compression effects become important. Airspeed indicators use such a device to measure airplane speeds relative to the air. Figure 48 is a photograph of a typical Pitot tube.

Figure 48.

Incidentally, if the flow is out of a system, either through the end of a pipe or from a hole in a container, then the velocity can be experimentally determined without using a pressure measurement. One has to measure the volume that escapes in a period of time. The volume which escapes can be computed using Figure 49. If the water flows at a speed v, then in a time t it would fill an imaginary cylinder whose base is A, the area of the hole whose length is vt. Thus the volume of outflow is $V = Avt$, or solving for the velocity:

$$v = \frac{V}{At}$$

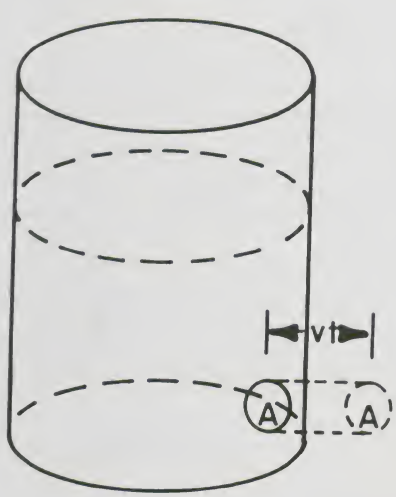


Figure 49.

Example 17. A vandal shoots a hole in a water tower with a 22-caliber bullet (diameter = .25 inches). If 0.43 ft^3 of water escapes in a minute, calculate the outflow velocity.

Solution. The area of the hole is $A = \pi D^2/4$.

$$A = \frac{3.14\,(.25)^2}{4} = .05 \text{ in}^2 = 3.6 \times 10^{-4} \text{ ft}^2$$

$$V = v_L A t$$

$$v_L = \frac{V}{At} = \frac{(0.43 \text{ ft}^3)}{(3.6 \times 10^4 \text{ ft}^2)(60 \text{ s})}$$

$$= 20 \text{ ft/s}$$

EXPERIMENT C-2. Blood Pressure Measurements

One familiar pressure measurement is arterial blood pressure, using a sphygmomanometer (sfig-mo-man-o-miter). This device (shown in Figure 50) enables a doctor or nurse to make an approximate measurement of the pressure in an artery *without* having access to the fluid (blood). It provides an indirect measure of the gauge pressure in the circulatory system. For medical purposes, easily reproducible information about abnormally high blood pressure or sudden changes in blood pressure is more important than extreme precision. Blood pressure actually depends in a complex way on heart muscle contraction, circulatory system constriction, location in the body, blood flow velocity, and many other factors. Difficulty with any one of a number of bodily functions may show up in an abnormal blood pressure.

Blood pressure measurement is an excellent example of a situation where the precise physics is complex, but a good understanding can come from an analogy with simpler hydraulic systems, such as those studied elsewhere in this module. Physicists often analyze complex systems in terms of simpler "models" they are able to understand more clearly.

Blood pressure varies between a maximum and a minimum value. The peak value, called *systolic pressure,* corresponds to the pumping action or contraction of the heart. Depending on the rate of heart contraction (pulse rate), the systolic pressure occurs about 70 times each minute (or once every 1/70 min). Figure 51 is a sketch of blood pressure (torr) versus time for a normal individual at rest. Notice that the time between peaks gives the pulse rate. The minimum pressure is called the *diastolic pressure,* and is never zero. These two values of pressure (maximum and minimum) are the variables recorded for a blood pressure measurement. A typical value for a healthy adult might be 130/80, which means that the systolic (maximum) pressure is 130 torr and the diastolic (minimum) pressure is 80 torr. The values found for a given person depend on many factors, including fatigue, excitement, anxiety, recent food, cigarettes or alcohol, and others. The most useful data are obtained under controlled conditions, when the person is at rest for a period of time prior to measurement. Often, repeated measurements are taken to improve accuracy.

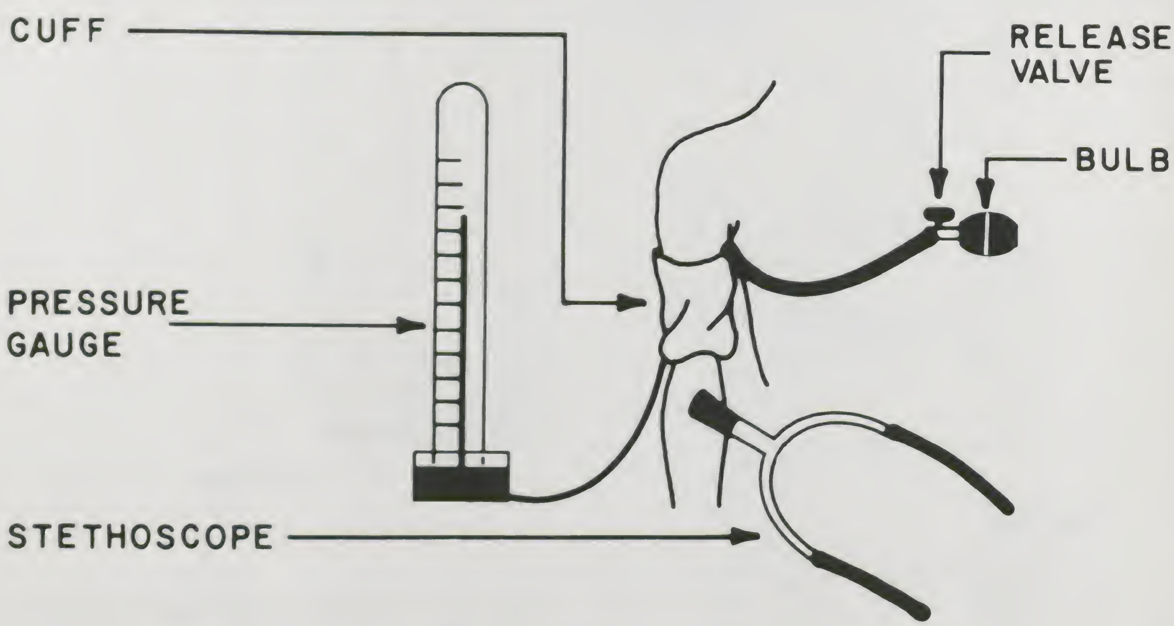


Figure 50. A sphygmomanometer.

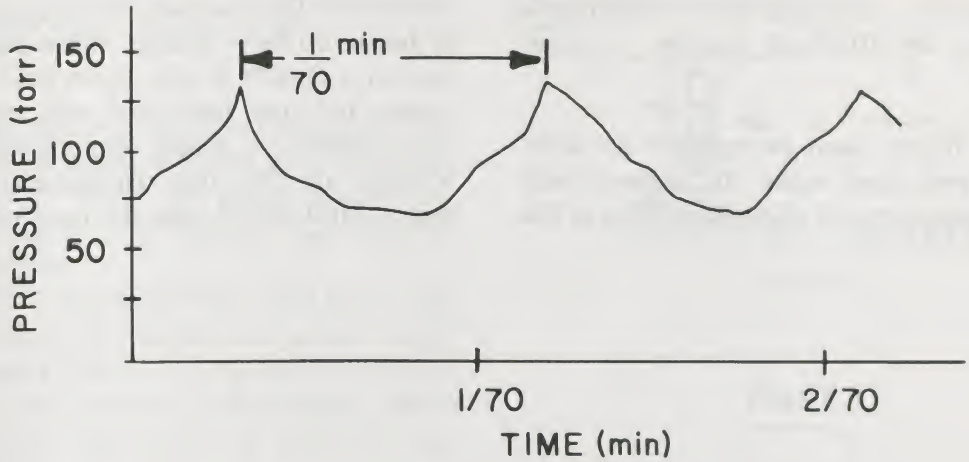


Figure 51.

PROCEDURE

Obtain a sphygmomanometer and identify the following parts (see Figure 50):

1. An inflatable cuff to wrap around the upper arm.

2. A pressure gauge (mercury manometer or Bourdon type) which indicates the pressure in the cuff in torr (or the equivalent older unit, millimeters of mercury).

3. A hand pump to inflate the cuff.

4. A valve to release air from the inflated cuff.

A stethoscope is required to use the sphygmomanometer for blood-pressure measurements. Blood pressure is measured by wrapping the inflatable cuff around the upper arm an inch or two above the elbow. The cuff is inflated to a pressure which is sufficient to constrict the main artery and thus stop the blood flow. Since the sphygmomanometer cuff, the skin on the arm, and the wall of the artery are all flexible, the pressure produced inside the cuff is transmitted to the inside of the artery. When the cuff pressure is greater than the arterial pressure, the artery is collapsed, stopping the flow of blood. Remember, however, that the arterial pressure varies with time between a maximum and a minimum value.

During measurement, the cuff pressure is slowly decreased (by letting air out of the cuff) until the pressure in the cuff, P_c, just equals (or is slightly less than) the systolic arterial pressure, P_b. At this cuff pressure some blood is forced through the constricted artery at each heart beat. The blood passing the constriction makes sound which can be detected with a stethoscope placed on the arm, just below the cuff (inside the elbow). At higher cuff pressures no sound is heard because no blood gets through. Thus, the determination of the systolic pressure corresponds to the cuff pressure at which fluid sounds are first heard, as the cuff pressure is slowly decreased from a value high enough to stop all blood flow.

By continuing to reduce the cuff pressure, the flow restriction is diminished. However, the stethoscope continues to pick up sounds as long as any constriction of the artery exists. Eventually the cuff pressure equals the minimum blood pressure and the constriction disappears. With no resistance to flow, the blood passes through without a sound. Thus, the cuff pressure when the sounds disappear is the diastolic pressure. This cuff pressure corresponds to the minimum blood pressure in an unconstricted artery.

Working in pairs, measure each other's blood pressure. Take repeated readings and don't expect great accuracy, since some prac-

tice is required. Look for left arm-right arm differences, the effect of exercise, or other variables.

Question. If you want to measure the *heart* pressure, you must make the measurement with the upper arm at chest level. Why is this necessary? Think about the simpler hydraulic systems you have studied. (What affects pressure in a fluid?) If you think you know the answer (or your instructor tells you), devise and perform a simple experiment to test whether or not this requirement is really important in blood pressure measurement.

HYDRAULICALLY ACTUATED CONTROLS

Liquid pressure and fluid flow are used to control mechanisms found in many common devices. Some of these devices are described below. You probably can think of other examples.

Figure 3 illustrated the float-level control in a toilet tank. As the float at one end of the lever rises with the water level, the other end of the lever presses a rubber-tipped piston against a valve seat, and this shuts off the water supply.

A carburetor float control (Figure 52) is quite similar. As the level of gasoline in the bowl of the carburetor drops, the float-lever mechanism opens the inlet from the fuel pump. What happens if the float is set too high or too low?

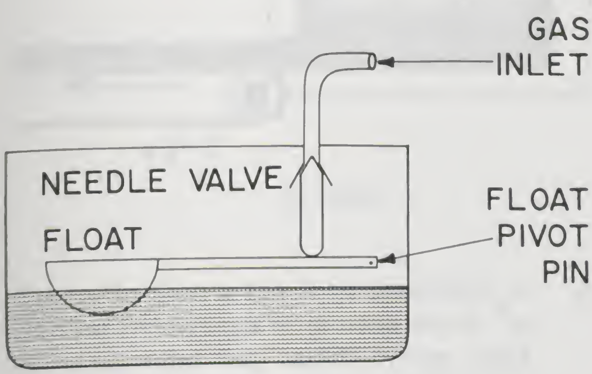


Figure 52.

An automobile radiator cap and a pressure cooker safety valve are safety devices which will open at a preset pressure. The pressure at which the radiator cap opens is determined by the stiffness of the spring, shown in Figure 53. In addition, pressure cookers have a pressure-regulation valve, as in Figure 54. The operating pressure of the cooker is determined by the weight of the valve. When the pressure becomes great enough, the weight is lifted and jiggles a bit, releasing some of the steam, thus keeping the pressure approximately constant.

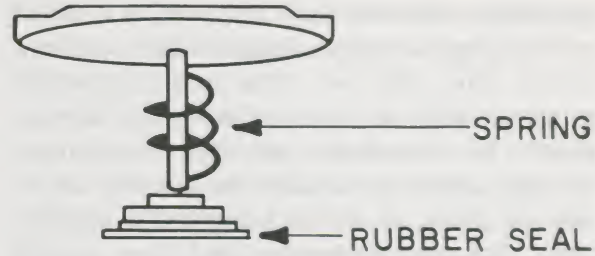


Figure 53.

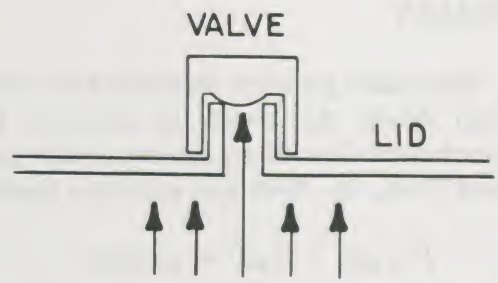


Figure 54.

The automobile brake-light switch and the oil-pressure indicator switch are good examples of pressure-activated electrical switches (see Figure 55). Pressure-activated switches of this type are connected to lights or buzzers to warn of high or low pressure conditions in hydraulic systems.

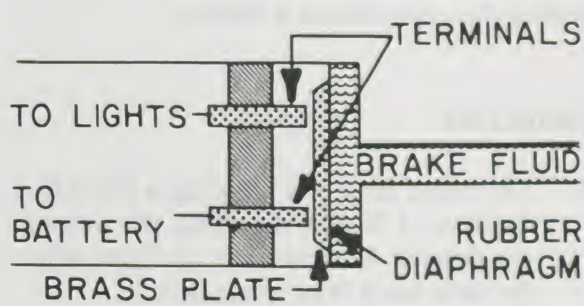


Figure 55.

These devices are specific examples of a type of technology called *fluidics*. This technology uses controlled fluid flow to perform the same control functions as do electronic devices. Fluidic devices are much slower than electronic devices, but they have one great

advantage. Because they are part of a sealed system, they are very reliable in dirty environments. Thus, they are often used in industrial processes such as metal refining and fabrication. In hospitals, portable respiratory therapy units use fluidics to control air or oxygen flow in assisted breathing applications. This use of pressure and flow control eliminates the need for a possibly dangerous electrical system in the apparatus.

SUMMARY

Bernoulli's principle states that the faster a fluid flows, the lower the pressure. For non-turbulent flow of an incompressible non-viscous fluid, the Bernoulli equation applies:

$$P + \rho g h + \tfrac{1}{2}\rho v^2 = \text{constant}$$

The aspirator is based on Bernoulli's principle.

The barometer, the manometer, the Bourdon gauge, and the Pitot tube are all used for pressure measurements.

The sphygmomanometer is used to measure blood pressure by determining the maximum and minimum pressures required to constrict an artery.

Atmospheric pressure and the variation of pressure with depth are the basic ideas behind the operation of a siphon.

PROBLEMS

1. The water tank of a building is filled to a depth of 4 ft. The air above the water is compressed to a pressure of 10 psi when the tank has 4 ft of water in it.

   a. What are the absolute and gauge pressures at the surface and at the bottom of the water?

   b. What is the velocity of water flow out of an open pipe at the bottom of the tank? (Assume that the tank is so large that for all practical purposes $v = 0$ in the tank.)

2. A river is 60 m wide and 3 m deep as it goes over a waterfall. The stream is flowing at 2 m. If the falls are 90 m high, how much energy would be available each second for hydroelectric power generation, if all the water could be channelled through a turbine?

3. A chocolate bar manufacturer wants to run liquid chocolate uniformly onto a moving conveyor belt. The belt moves at a speed of 8 ft/s. What height of chocolate in the storage tank will maintain the proper outflow onto the moving belt? (You must make the assumption that the flow is frictionless.)

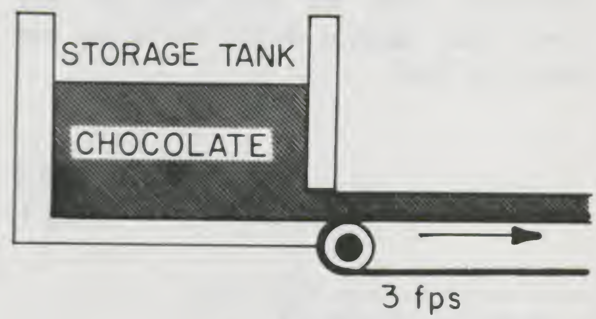


Figure 56.

4. A swimming pool has a hole of area ½ in^2 at the bottom. Water freely runs out. How many gallons per hour must be supplied to maintain the water level at 4 ft above the bottom?

5. A small town (area 3.5 square miles) has a single 16-in-diameter storm sewer pipe which drains rain water from the town to a river at the bottom of a canyon 60 ft below the town's level. If 1.3 in of rain falls in 2 h and 40% is lost to ground absorption and evaporation, how long will it take for all the rainfall run-off to be cleared through the sewer?

6. A stream of water with a 1-in diameter is projected with a velocity of 20 ft/s against a wall. What pressure would a gauge indicate at the point of impact?

7. Water pressure in a house is 30 lb/in^2. A leaky faucet drips 1 gal per h. What is the effective area of the leak?

8. A test chamber at a submarine design laboratory operates to simulate the sub's maximum speed by flowing water past a fixed model of the submarine. The pressure difference measured with a Pitot tube is 20 lb/in^2.

   a. Calculate the water flow velocity (submarine's maximum speed).

   b. If the test tank flows 100,000 gal in 90 s and the test chamber is 3 ft^2 in cross-sectional area, recalculate the flow velocity and compare your answer to the result in part a.

WORKSHEET

EXPERIMENT A-1. Devices

A. Auto Jack

1. Student's weight (approximate)

_______________________________ .

Applied force (read on spring scale)

_______________________________ .

$$\frac{\text{Weight}}{\text{Force}} = \_\_\_\_\_\_\_\_ = \_\_\_\_\_\_\_\_$$

2. Description of jack:

____

____

____

____

____

____

____

____

____

____

____

3. Length of handle, from pivot to end:

L_1 = ________

Distance from pivot to small cylinder:

L_2 = ________

Ratio:

$$\frac{L_1}{L_2} = \_\_\_\_\_\_\_\_ = \_\_\_\_\_\_\_\_$$

C. Questions you would like answered about the hydraulic devices in the lab:

1. _______________________________

____

____

2. _______________________________

____

____

3. _______________________________

____

____

4. _______________________________

____

____

WORKSHEET

EXPERIMENT A-2. Hydrometry

A. 1. Known Liquids

Liquid	Known Specific Gravity	Measured Specific Gravity

2. Unknown Liquids

Liquid	Specific Gravity	Identification of Liquid
A		
B		
C		
D		
E		

3. Sugar Solution
Range of Specific Gravity.

From ________ to ________

B. 1. Density of the balls in an inexpensive hydrometer.

Volume H_2O (cm^3)	Volume Antifreeze (cm^3)	Number of Balls Which Float	ρ_{mix} (g/cm^3)
50	0		
50	5		
50	10		
50	15		
50	20		
50	25		
50	30		

3. Comparison of experimental values for freezing points of antifreeze solution to freezing point indicated by container label.

Protection to (°F)	Percentage Antifreeze Experimental Value	Percentage Antifreeze Value from Container
+30°		
+15°		
0°		
-15°		

WORKSHEET

EXPERIMENT B-1. Pressure Dependence on Depth

1, 2. Data Table

Port	Spring Balance Reading (g)	Force (dyn)	Depth (cm)
1			
2			
3			
4			

4. Slope of the Graph

a. From your graph, find the values of pressure and depth at two points (use two points on the line you draw, not two actual data points):

P_1 = ________ (dyn/cm^2)

P_2 = ________ (dyn/cm^2)

h_1 = ________ (cm)

h_2 = ________ (cm)

b. Calculate the rise and the run:

Rise = $P_2 - P_1$ = ________

Run = $h_2 - h_1$ = ________

c. Calculate the slope:

$$\text{Slope} = \frac{\text{rise}}{\text{run}} = \frac{P_2 - P_1}{h_2 - h_1}$$

WORKSHEET

EXPERIMENT B-2. Measuring Density

A. Finding Density from Displaced Volume

Mass of Object (g)	Volume of Water in Graduated Cylinder (cm^3)	Volume of Water plus Object (cm^3)	Volume of Object (cm^3)	Density of Object (g/cm^3)

B. A More Accurate Measure of Density

	Object No. 1	Object No. 2
Mass in air M_A (g)		
Apparent mass in water M_W (g)		
Volume $(M_A - M_W)/(1\ g/cm^3)$ (cm^3)		
Density ρ_x (g/cm^3)		

C. Objects Which Float

	Object No. 1	Object No. 2
Mass of object in air M_A (g)		
Apparent mass Object in air plus sinker in water $(M_A + M_{SW})$ (g)		
Apparent mass Both object and sinker in water $(M_W + M_{SW})$ (g)		
Volume of object $[(M_A + M_{SW}) - (M_W + M_{SW})] / (1 \text{ g/cm}^3)$		
Density ρ_x (g/cm^3)		

WORKSHEET

EXPERIMENT B-3. A Closed Hydraulic System

1. Diameter of large plunger ________ .

   Diameter of small plunger ________ .

2.

Input Force Mg (dynes)	Output Force Before Tapping	Output Force After Tapping	Pressure at Large Plunger	Pressure at Small Plunger	Ratio of F_{out}/F_{in}

Average ratio ________

0-07-001724-7